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A Probabilistic Storm Transposition Approach for Estimating Exceedance Probabilities of Extreme Precipitation Depths

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A storm transposition approach is investigated as a possible tool of assessing the frequency of extreme precipitation depths, that is, depths of return period much greater than 100 years. This paper focuses on estimation of the annual exceedance probability of extreme average precipitation depths over a catchment. The probabilistic storm transposition methodology is presented, and the several conceptual and methodological difficulties arising in this approach are identified. The method is implemented and is partially evaluated by means of a semihypothetical example involving extreme midwestern storms and two hypothetical catchments (of 100 and 1000 mi^2 ($\approx$260 and 2600 km^2)) located in central Iowa. The results point out the need for further research to fully explore the potential of this approach as a tool for assessing the probabilities of rare storms, and eventually floods, a necessary element of risk-based analysis and design of large hydraulic structures.

1. Introduction and Problem Statement

Current design standards for major hydraulic structures in the United States and other countries are based on the deterministic probable maximum flood (PMF) procedure. The PMF is defined as the flood resulting from the "extreme application" over the basin of the probable maximum precipitation (PMP). PMP is the "theoretically greatest depth of precipitation for a given duration that is physically possible over a particular drainage area at a certain time of the year" [*American Meteorological Society*, 1959, p. 446]. A revised definition of PMP makes the distinction that PMP is a function of storm area, as opposed to the 1959 definition relating it to drainage area [*Hansen et al.*, 1982]. The details of the PMP estimation procedure are described in several publications (*Myers* [1967], *World Meteorological Organization* [1973], and *Hansen* [1986*a*], among others).

The traditional approach to PMP estimation is deterministic and therefore does not provide any estimate of risk. With the movement, however, toward risk-based engineering design [e.g., *American Society of Civil Engineers*, 1973; *National Research Council*, 1983, 1985, 1988; *Dawdy and Lettenmaier*, 1987], the question often asked is the following: What is the probability of exceeding the PMP/PMF estimates? In this paper a method of assigning probabilities to extreme precipitation depths (recurrence interval greater than 100 years up to near PMP depths) is formulated and studied. The method is based on a frequency analysis of the regional characteristics of extreme storms and a probabilistic approach to storm transposition.

The concept of storm transposition has been used extensively in a deterministic framework for the derivation of PMP estimates [e.g., *Schreiner and Riedel*, 1978; *Hansen et al.*, 1982] but has not been rigorously investigated in a probabilistic framework. To the best of my knowledge the few studies addressing this problem are those of *Alexander* [1963, 1969], *Gupta* [1972], *Newton* [1983], *Yankee Atomic Electric Company (YAEC)* [1984], and *Fontaine and Potter* [1989], with the study of *Gupta* [1972] offering the most formal mathematical presentation. These studies, some reviewed in the recent *National Research Council Report (NRC)* [1988], have several methodological similarities but also distinct procedural dissimilarities. For example, *Gupta* [1972] and *Fontaine and Potter* [1989] transpose historical storms, while *Alexander* [1963] and *YAEC* [1984] transpose conceptualized single-center storms with isohyetal patterns derived from the maximum depth-area-duration (DAD) curves. Alexander and YAEC have made several arbitrary simplifying assumptions [see, *NRC*, 1988, chapter 4] in accounting for the storm/catchment interactions. The storm/catchment interaction, however, is an important element of the whole analysis since it directly affects the probability distribution of the total rainfall depth over the catchment and the distribution of the catchment area wetted by the storm or covered by a depth exceeding a specified value.

One of the main difficulties of assessing the probability of rare storms stems from the complicated multivariate stochastic structure of rare storms, which is difficult to assess since by definition only a few such rare events have been observed. Also, how should one attach a frequency to a whole storm, that is, which random variable(s) should be used for a frequency analysis? Possible candidates are the following: the average depth over the total storm duration and total storm areal extent, the maximum depth at the storm center, the average maximum depth over an area equal to the area of the catchment of interest, the storm duration, the temporal distribution of the rainfall depths, the storm orientation, the storm areal extent, etc. Moreover, how should one define the meteorologically or statistically homogeneous area within which storms can be transposed?

For risk-based design the ultimate interest lies in the estimation of exceedance probabilities of extreme floods. It is imperative, however, that before one proceeds to the complex rainfall-runoff relationships the merits of the probabilistic storm transposition approach have been rigorously investigated and proven promising at the first level, that is, at the level of reliable estimation of exceedance probabilities of extreme precipitation depths over a catchment. Despite the general agreement on the need of such estimates, efforts to date on the probabilistic storm transposition approach have been sporadic and unconnected, partially due to the lack of a suitable analysis framework. The aim of this paper is to

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present such a framework and set forth the several conceptual and methodological difficulties arising in the probabilistic storm transposition approach. A preliminary implementation of the developed approach is also presented, and the tail of the probability distribution of the maximum annual average depth over two hypothetical catchments (of 100 and 1000 mi² (≈260 and 2600 km²)) located in Iowa is estimated.

2. General Formulation of the Probabilistic Storm Transposition Approach

In the description of the general framework that follows, several new concepts and variables are introduced but are left purposely vague. Upon completion of the methodology a detailed elaboration on the introduced concepts and variables is given.

Let $\mathcal{S}$ denote the set of all conceivable storms and $\mathcal{S}_E$ a subset of $\mathcal{S}$ composed of "extreme" storms only. Specifically, $\mathcal{S}_E$ consists of all storms that satisfy a given criterion E of storm severity. The selection of criterion E is arbitrary; it reflects the interest in extreme storms only since moderate storms would not provide any information on the distribution of extremes, that is, events of return period greater than 100 years.

Let Λ_s denote the random vector of storm characteristics describing a storm. In general, Λ_s will be comprised of the parameters of a stochastic model describing the rainfall field. Depending on the model, these parameters may or may not be directly interpretable in terms of physical storm characteristics. Let Λ_p denote the two-dimensional vector describing the position of a storm (here this position is called storm center). The storm center may be defined as the location of the maximum observed total depth or as the location of the maximum accumulated depth over a period of time. Alternatively, it may be defined as the center of mass of the storm.

Let $Z(t)$ denote the number of extreme, that is, satisfying criterion E, storm occurrences in an interval of t years (stationarity in time is assumed) and $\bar{d}_c(\Delta t)$ denote the average rainfall depth deposited from a particular storm over the catchment during a time interval Δt. Assuming that the number of extreme storms in a year is independent of the random variable $\bar{d}_c(\Delta t)$ and that the average depths $\bar{d}_c(\Delta t)$ produced by different storms are mutually independent identically distributed, the annual probability of exceedance of $\bar{d}_c(\Delta t)$ can be written as

$$p_a(\bar{d}_c(\Delta t) \geq d) = 1 - \sum_{\nu=0}^{\infty} p[Z(1) = \nu] \cdot I^{\nu} \tag{1}$$

where

$$I = \int_{[\Lambda_s]} \int_{[\Lambda_p]} p(\bar{d}_c(\Delta t) < d|\lambda_s, \lambda_p) f(\lambda_s, \lambda_p)\, d\lambda_s\, d\lambda_p \tag{2}$$

In the above equation, $[\Lambda]$ denotes the functional space (range) within which the vector Λ varies and $f(\lambda_s, \lambda_p)$ denotes the joint probability distribution of the random vectors Λ_s and Λ_p, which represent storm characteristics and storm location. The range of storm characteristics Λ_s is bounded from below because of the restricted interest in extreme storms, that is, storms satisfying the storm severity criterion E. The range of storm positions Λ_p is restricted to locations surrounding the catchment and, in particular, to locations within the effective area (A_{eff}) of the catchment. The effective area is defined as that area within which if the storm center Λ_p falls, it will produce at least one point on the catchment with nonzero rainfall depth. Obviously, A_{eff} depends on the shape of the catchment and the shape of the storm. For example, for a circular catchment of radius r_c and a circular storm with radius r_s, the effective area is a homocentric circle of radius $(r_c + r_s)$. Depending on the shape of the storm and the shape of the catchment, this effective area will have a shape which may not be easily described geometrically.

The joint probability distribution $f(\lambda_p, \lambda_s)$ can be analyzed either as $f(\lambda_p|\lambda_s)\, f(\lambda_s)$ or as $f(\lambda_s|\lambda_p)\, f(\lambda_p)$. The conditional probability $f(\lambda_p|\lambda_s)$ denotes the probability that a storm of given characteristics λ_s will occur at position λ_p, while the conditional probability $f(\lambda_s|\lambda_p)$ denotes the probability that a storm that has occurred at position λ_p will have characteristics λ_s. Although both avenues of decomposition are possible, we choose to proceed here with the first approach because $f(\lambda_p|\lambda_s)$ is easily interpreted in terms of storm transposition probabilities. In view of this, (2) is further written as

$$I = \int_{[\Lambda_s]} \int_{[\Lambda_p]} [1 - p(\bar{d}_c \geq d|\lambda_s, \lambda_p)]\, f(\lambda_p|\lambda_s) f(\lambda_s)\, d\lambda_s\, d\lambda_p \tag{3}$$

$$I = \int_{[\Lambda_s]} f(\lambda_s) \cdot \left\{ \int_{[\Lambda_p]} [1 - p(\bar{d}_c \geq d|\lambda_s, \lambda_p)] f(\lambda_p|\lambda_s \right)\, d\lambda_p\, d\lambda_s \tag{4}$$

where $\bar{d}_c(\Delta t)$ has been abbreviated to $\bar{d}_c$.

The stochastic storm transposition approach exhibits difficult estimation problems, especially the estimation of $f(\lambda_s, \lambda_p)$. In the following sections an attempt is made to define these problems and offer a framework of analysis.

3. Transposition Area A_{tr} and Nonhomogeneous Storm Transposition Probabilities

The probability $f(\lambda_s)$ in (4) cannot be reliably estimated from the very few extreme storms that have occurred over the catchment of interest. If regional characteristics are taken into consideration, however, the set of available storms can be enlarged to include all storms satisfying criterion E of storm severity and having their centers within a major area of similar regional characteristics (substitution of space for time). This major area is called here storm transposition area A_{tr} (see Figure 1). The storm transposition area is the area within which all the occurred storms can be transposed anywhere with an adjustment to their probability of occurrence (or, alternatively, could be transposed anywhere with the same occurrence probability but with an adjustment to their depths). It is desirable that the transposition area does not exhibit pronounced meteorological or topographical anomalies since this complicates the reliable estimation of the transposition probabilities. It is realized that if all storms had equal chance of occurring anywhere within the area A_{tr}, then the extreme rainfall depth estimates for a fixed return period would be equal at all locations

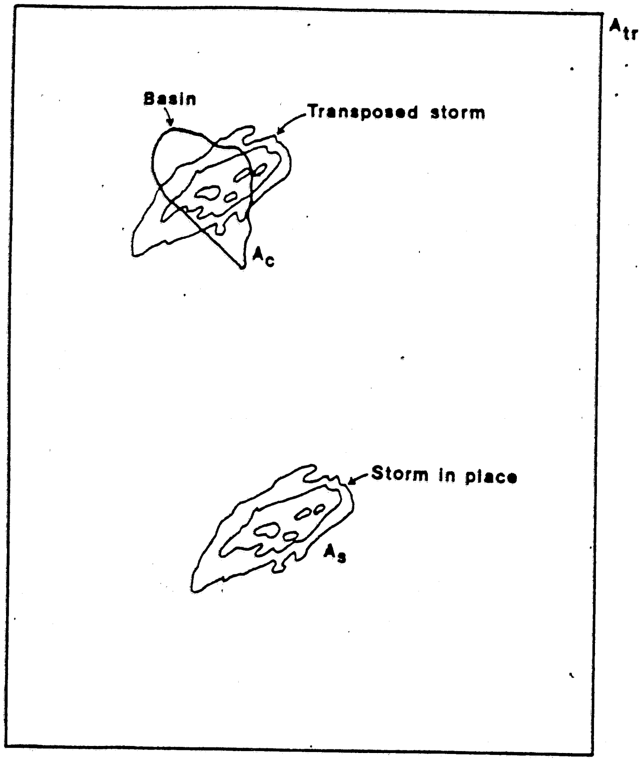

Fig. 1. Schematic representation of the storm transposition procedure.

within this area. It is known, however, that considerable spatial variability in extreme precipitation depth estimates does exist, as depicted in Figure 2 for the midwest [e.g., *Hershfield*, 1961; *Schreiner and Riedel*, 1978]. This spatial variability can be viewed as the result of differences in regional storm characteristics and can be incorporated in the developed framework by considering variable probabilities of storm transposition. The estimation of storm transposition probabilities $f(\lambda_p|\lambda_s)$ is a difficult problem which has not been addressed in previous studies and which has only preliminary discussion in this paper. In particular, we offer here some general thoughts connecting this problem to a marked or multivariate spatial point process framework.

In principle, the occurrence of storms in a major area R is a marked point process where the mark is a random vector composed of continuous random variables describing several storm characteristics. As an example, consider the major area R of the nine midwest states (Figure 3), and suppose that the magnitude of a storm is described by the total maximum storm depth d_0, that is, the maximum depth deposited by the storm over its total duration. Storm depth d_0 is a continuous random variable with a probability distribution which can be characterized on the basis of a frequency analysis of available data. The spatial occurrence of storms described by d_0 is a marked spatial point process. Because of interest in extreme storms only, d_0 is truncated from below by a storm depth equal to $d_{\min}$, that is, only storms for which $d_0 \geq d_{\min}$ are considered. If one discretizes the continuous random variable d_0 into m ranges $[d_{\min}, d_1)$, $[d_1, d_2), \cdots, [d_{j-1}, d_j), \cdots, [d_{m-1}, d_m)$, then one can think of an event as being of type j if $d_0 \in [d_{j-1}, d_j)$. This gives rise to a multivariate spatial point process (see, for example, *Daley and Vere-Jones* [1988]).

For illustration purposes, consider an arbitrarily selected cutoff level equal to 9 in (22.9 cm). If one considers three ranges of d_0, for example, 9 in (22.9 cm) $\leq d_0 <$ 10.5 in (26.7 cm); 10.5 in (26.7 cm) $\leq d_0 <$ 13.0 in (33 cm); and $d_0 \geq$ 13 inches (33 cm), one can view the storms as falling in one of the following three types:

Type 1 storms

$$9.0 \leq d_0 < 10.5 \text{ in}$$

Type 2 storms

$$10.5 \leq d_0 < 13.0 \text{ in}$$

Type 3 storms

$$13.0 \leq d_0 \text{ in}$$

On the basis of the extreme storm data of the *U.S. Army Corps of Engineers* [1948] catalog and for the period of 1891 to 1951, 33 storms were of type 1, 26 of type 2, and 18 of type 3. Figures 3*a*, 3*b*, and 3*c* show the spatial occurrence process of each one of these types of events (marginal point processes), and Figure 3*d* shows the pooled point process of storm occurrences as the superposition of the three marginal point processes. Obviously, the marginal and pooled spatial point processes of storm occurrences are not homogeneous over the area R, that is, their intensities are not constant but

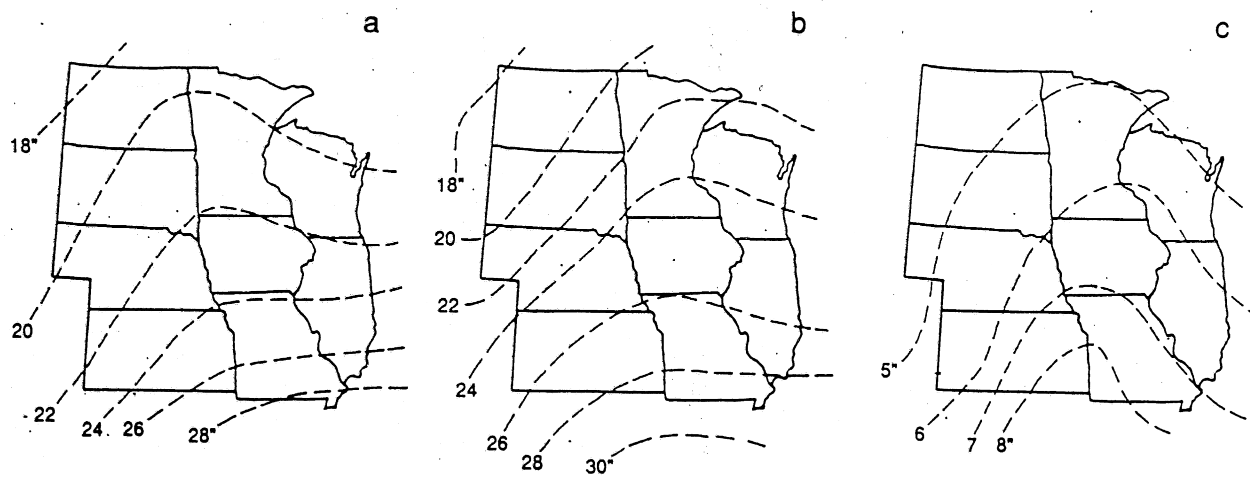

Fig. 2. Spatial variability of extreme precipitation depths in the midwest. (a) 24-hour, 200 mi² PMP, (b) 6-hour, 10 mi² PMP, and (c) 100-year, 24-hour, 10 mi² rainfall depth. One square mile equals 2.6 km².

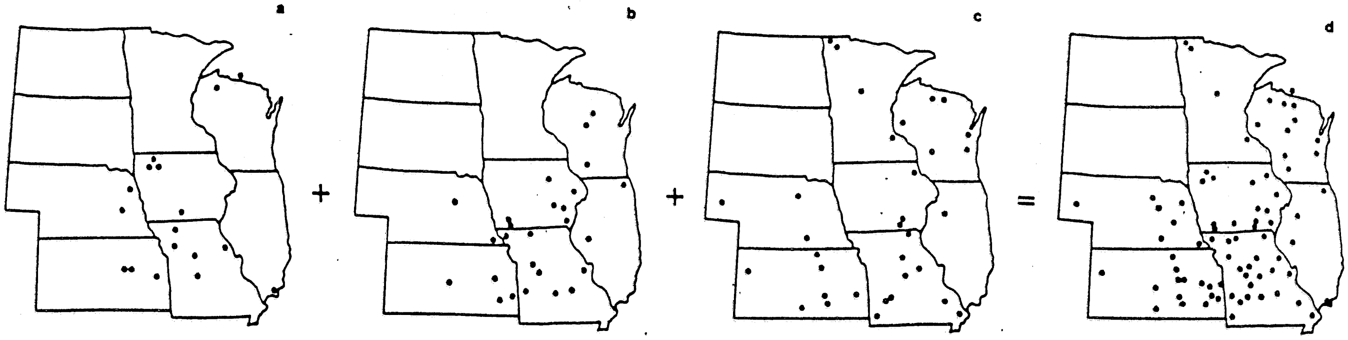

Fig. 3. Three marginal point processes of storm occurrences and the pooled multivariate point process of all storms. (a) Point process of type 1 storms (9 in $\le d_0 <$ 10.5 in); (b) point process of type 2 storms (10.5 in $\le d_0 <$ 13 in); (c) point process of type 3 storms (13 in $\le d_0$); and (d) multivariate point process of all storms satisfying criterion E ($d_0 \ge$ 9 in).

depend on the position $\mathbf{x}$. For a marked point process the first-order intensity function may be defined as

$$\lambda(\mathbf{x}, \mathbf{m}) = \lim_{|d\mathbf{x}| \to 0} \frac{P[N(d\mathbf{x}) > 0, \mathbf{M}(\mathbf{x}) \ge \mathbf{m}]}{|d\mathbf{x}|} \quad (5)$$

where $N(A)$ is the number of events in an area A, $d\mathbf{x}$ is a small area located at the spatial position $\mathbf{x}$, and $\mathbf{M}(\mathbf{x})$ is the magnitude of an event at location $\mathbf{x}$. The first-order intensity, or rate of occurrence, denotes the probability of having an event of magnitude greater or equal to $\mathbf{m}$ at location $\mathbf{x}$. If the rate of occurrence of the marked point process were known, then, in principle, one could estimate $f(\boldsymbol{\lambda}_s, \boldsymbol{\lambda}_p)$ and, consequently, the exceedance probability of rainfall depths over any area of a given size surrounding point $\mathbf{x}$ by integrating over the appropriate range of storm locations and storm magnitudes (equations (1) and (2)). The storm transposition probability is, in essence, a conditional rate of occurrence $\lambda(\mathbf{x}|\mathbf{m}_j)$, where $\lambda(\mathbf{x}|\mathbf{m}_j)$ denotes the probability with which an event of magnitude greater or equal to $\mathbf{m}_j$, which actually occurred at location $\mathbf{x}_j$, could occur at position $\mathbf{x}$.

Estimation of marked or multivariate spatial point processes is difficult in general [e.g., *Kutoyants*, 1984]. It becomes even more difficult in the case of extreme storms because of limited data and the complexity of defining the descriptive vector (mark) of each storm. If areal extent and shape of storms are included in the descriptive vector, then principles of stochastic geometry [e.g., *Stoyan et al.*, 1987] may be useful in the estimation.

4. Criterion of Storm Severity

The selection of criterion E is arbitrary. For example, if $d[\Delta t, A]$ denotes the storm depth accumulated over a period Δt and averaged over an area A, one could select

Criterion E

$$\{d[\Delta t, A] \ge d_{\min}\} \quad (6)$$

where Δt could be a fixed time period of 6, 12, or 24 hours, etc., or equal to the total storm duration, or approximately equal to the time of concentration of the basin, depending on the hydrologic application. In addition, the time period Δt might be considered as the period $(0, \Delta t)$, that is, from the beginning of the storm or as the period $\Delta\bar{t} = (t_s, t_s + \Delta t)$, where t_s is defined as

$$\int_{t_s}^{t_s+\Delta t} i(\tau)\, d\tau \ge \int_{t}^{t+\Delta t} i(\tau)\, d\tau \quad \forall\, t \ge 0 \quad (7)$$

and where $i(t)$ is the rainfall intensity at time t. In other words, $\Delta\bar{t}$ is defined as the interval over which the cumulative depth is maximum as compared to all other depths accumulated over time periods of equal length Δt. The use of the time period $\Delta\bar{t}$ instead of Δt is common in extreme rainfall analysis, as, for example, for the construction of the intensity-duration-frequency curves used for hydrologic and hydraulic design of small structures (design events of return period less than or equal to 100 years).

Similarly, the size of area A may be a fixed area of size 100, 200 mi^2 ($\approx$260, 520 km^2), etc. or approximately equal to the size of the catchment of interest. Also, A may be considered as that area of the storm over which the average depth is maximum as compared to the average depth over any other area of equal extent. In that case $A = \bar{A}$, where $\bar{A}$ is defined as

$$\int_x\int_y\limits_{(x,y)\in\bar{A}} d(\Delta t, x, y)dx\, dy \ge \int_x\int_y\limits_{(x,y)\in A} d(\Delta t, x, y)\, dx\, dy \quad \forall A \quad (8)$$

where $d(\Delta t, x, y)$ is the storm depth accumulated during a period Δt over the point of spatial coordinates (x, y). The use of area $\bar{A}$ is common in extreme storm analysis, as, for example, in the estimation of the depth-area-duration curves used for the reconstruction of design storms.

The selection of criterion E affects the set $\mathcal{S}_E$ of storms available for estimation and thus may affect the final estimates of exceedance probabilities of extreme rainfall depths. Obviously, this is not desirable, and the final estimates must be independent of the selected criterion E. At the same time, criterion E should be efficient in the sense that reliable estimates should be obtained with the transposition of a minimum number of extreme storms. An important additional reason for selecting Criterion E as large as possible is that it provides more assurance that the available set of storms $\hat{\mathcal{S}}_E$ will be a representative sample of the true population set $\mathcal{S}_E$, since the most extreme storms are more likely to have been recorded and documented. (Information on the catalog of extreme storms available for the estimation is given in section 6.) The use of thresholds for flood quantile estimation has been studied by *Smith* [1987].

5. STORM DESCRIPTOR Λ_s AND EXTREME RAINFALL MODELS

Ideally, a storm is described by the three-dimensional continuous function $d(t, x, y)$, giving the storm depth at any location (x, y) at any time t, $t \in [0, t_r]$, where t_r is the storm duration. For a probabilistic analysis of extreme catchment depths, simpler storm descriptions may be sufficient. For example, one may consider the storm as stationary, described by the spatial distribution of the cumulative precipitation depths during a time period Δt, where Δt is approximately equal to the time of concentration of the catchment of interest. As discussed above, $\Delta \bar{t} = (t_s, t_s + \Delta t)$, where t_s is defined in (7), is a possible choice. After the time period has been decided upon the storm is completely described by the two-dimensional spatial distribution function $d(x, y)$. Several stochastic spatial rainfall models for $d(x, y)$ exist in the literature [e.g., *Rodriguez-Iturbe et al.*, 1987; *Smith and Karr*, 1988]. These models have been mainly developed for rainfall events less extreme than the events considered here, and their performance, in terms of preserving extreme rainfall characteristics, has not been fully tested yet. This fact, together with the lack of detailed data for extreme rainstorms and the still exploratory stage of the storm transposition approach, dictates at present the use of simpler models and especially models with few parameters, which can be easily interpreted in terms of physical storm characteristics and which can be estimated from lumped data such as the published depth-area-duration data. A class of such simple models is described below.

Assuming that the storm isohyetal pattern is single centered and the contours are elliptical and geometrically similar, the depth at a point described by the polar coordinates (r, θ) relative to the storm center can be written in the form

$$d(r, \theta) = f(r, \theta, \boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a set of parameters constant for the whole storm and $d(r, \theta)$ is the depth at point (r, θ), where r is the distance from the storm center and the angle θ is measured counterclockwise from the major axis of the elliptical storm (Figure 4). In the developments that follow, and throughout the paper, we denote by $\bar{d}(A)$ the average depth of a storm over an area A and by $d(A)$ the value of the depth along the isohyet enclosing an area A; note that for the same area the average depth is always greater than the exceedance depth, that is, $\bar{d}(A) \geq d(A)$.

Horton [1924] observed that for many storms $\bar{d}(A)$ was related to the size of area A by a relationship of the form

$$\bar{d}(A) = d_0 \exp\,(-k|A|^n) \tag{9}$$

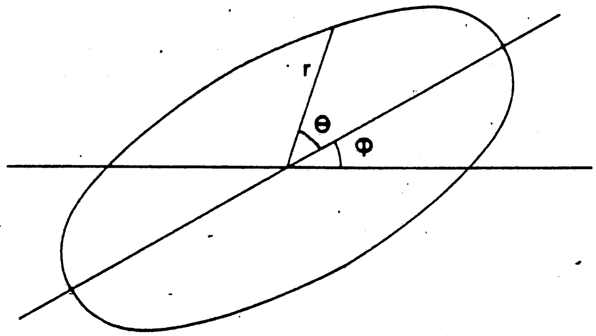

Fig. 4. Elliptical shape storm and definition of parameters r, θ, and ϕ.

where d_0 is the depth at the storm center and k and n are model parameters. From this equation the depth along an isohyet enclosing an area A is

$$d(A) = \frac{dV}{dA} = \frac{d[\bar{d}(A)|A|]}{dA} = d_0 e^{-k|A|^n}\,(1 - kn|A|^n) \tag{10}$$

For a circular storm the depth along an isohyet at distance r from the center is

$$d(r) = d_0[1 - kn\pi^n r^{2n}] \exp\,(-k\pi^n r^{2n}) \tag{11}$$

For an elliptical storm of minor to major axis ratio equal to c this relationship can be shown to be

$$d(r, \theta) = d_0 \exp\,[-\xi(r, \theta)][1 - n\xi(r, \theta)] \tag{12a}$$

where

$$\xi(r, \theta) = k(\pi/c)^n\,(\sin^2\theta + c^2\cos^2\theta)^n r^{2n} \tag{12b}$$

Horton [1924], *Court* [1961], and *Boyer* [1957], give ranges of k and n for several types of storms reported in the literature. Court and Boyer list several other expressions for the spatial distribution of rainfall within a storm.

For extreme storms in Illinois, *Huff et al.* [1958] and *Huff and Semonin* [1960] used the logarithmic relationship

$$\log \bar{d}(A) = a + b|A|^n \tag{13}$$

where a, b, and n are parameters to be estimated. For an elliptical storm one obtains from (13)

$$d(r, \theta) = 10^{a + b\zeta(r,\theta)}\{1 + n\,b\zeta(r, \theta)\ln 10\} \tag{14a}$$

where

$$\zeta(r, \theta) = (\pi/c)^n r^{2n}(\sin^2\theta + c^2\cos^2\theta)^n \tag{14b}$$

The above two models, (9) and (13), give isohyetal patterns of the same decaying form. The difference lies in the fact that the first model explicitly preserves the maximum observed depth while the second model has the flexibility of not preserving it explicitly and of assigning any nonzero area to the maximum observed depth. This flexibility is desirable since the maximum observed depth is not necessarily equal to the actual maximum depth. In fact, these depths can be quite different, depending on the storm isohyetal pattern and the raingage network [*Foufoula-Georgiou*, 1989].

On the basis of the above models and letting ϕ denote the storm orientation, the storm descriptor Λ_s for model (9) would be $\Lambda_s = (\phi, c, d_0, k, n)$ and for model (13) would be $\Lambda_s = (\phi, c, a, b, n)$. The joint probability density function of Λ_s needed in equation (4) can be estimated from a statistical analysis of the regional storm characteristics within the storm transposition area. Such an analysis has been recently presented by *Foufoula-Georgiou and Wilson* [1989].

6. CATALOG OF EXTREME STORMS

Extreme storms have been recorded since 1819, and there exists a data base of a total of 853 storms in the contiguous United States. The data collection and processing was a joint effort of the U.S. Army Corps of Engineers and the U.S. Weather Bureau. From the 853 storms, 314 storms are either incomplete, or the depth-area-duration (DAD) analysis is not considered very exact [*Shipe and Riedel*, 1976]. The most complete and accurate part of the data base consists of 539 storms published in a report by the *U.S. Army Corps of Engineers* [1948]. Each storm is described in two typical sheets, including information such as the date, duration and location of the storm, the total storm isohyetal maps, DAD tables for durations of 6, 12, 18, 24, . . ., hours, and mass curves for selected stations. Additional supporting data are on file at the National Weather Service. This data base has been mainly used to derive estimates of probable maximum precipitation. It provides a unique source of information about extreme storms, due to its length and extensive areal coverage. This information has not been adequately explored from the probabilistic point of view in terms of estimation of return periods of extreme precipitation depths.

Since the criteria used by the U.S. Army Corps of Engineers [1948] for including a storm in the catalog are not well defined and may have changed over the years, the set of available storms $\hat{\mathcal{S}}_E$ may not always be a representative sample of the true set $\mathcal{S}_E$. Preliminary analysis of extreme midwestern storms [*Foufoula-Georgiou and Wilson*, 1989] suggests that the extreme storm catalog in the midwest may not be complete for storms with a total maximum point storm depth less than 7 in (17.8 cm). Conclusions about the completeness of the record for more extreme storms, or for other regions in the United States is premature. A statistical analysis of temporal and spatial storm occurrences in conjunction with theory of thinned point processes (i.e., point processes with randomly or systematically deleted events) may be used to study storm record incompleteness. Such structured approaches of studying record incompleteness, and accounting for that in the estimation, have been developed in the context of earthquake estimation [*Veneziano and Van Dyck*, 1985, 1987]. Similar procedures need to be studied and implemented for the problem of extreme storm estimation.

Another problem accounted in the extreme storm catalog is that of accuracy in estimation of the storm peak, which is often used for frequency analysis and classification of storms. For example, the true storm maximum is never measured, but it is approximated by the maximum recorded amount at a station close to the true maximum. The difference between the true maximum and recorded maximum is a function of the storm spatial pattern and rain gage density. In storm transposition studies it is important to account for the different accuracy of each storm, since rain gage networks have changed over time and are of different density from place to place. In general, one needs to assess the error of the maximum recorded value of a random field, given the stochastic structure of the random field and the recording network. This is a difficult problem for which analytical results are not easy to obtain, and only asymptotic approximations may be possible [e.g., *Vanmarcke*, 1983, chapter 4], or it can be approached via simulation. Given, however, that within the core of the storm the storm pattern is usually a single-center well-behaved one, *Foufoula-Georgiou* [1989] presented a simplified error analysis which considers the rainfall field as being described by a deterministic spread function of a known functional form and parameters but with a center occurring randomly within the rain gage network. This analysis seemed to reasonably approximate the error in the maximum recorded depth, as evidenced by comparisons with experimental results of *Huff and Semonin* [1960]. Accuracy of extreme rainstorm records and its effect on the estimates of extreme precipitation depths and floods has been discussed in several studies [e.g., *Richards et al.*, 1988], but it has not been studied systematically within the framework of stochastic storm transposition.

7. A Simplified Implementation in the Midwest

In the previous sections the probabilistic storm transposition methodology was developed, and the problems which need further research were formulated. Although much theoretical work remains to be done before the method can be used for reliable estimation of extreme precipitation depths, an implementation of the method at this point is very illustrative and provides insight on the sensitivity of the estimates to several modeling and estimation assumptions, dictated by data availability. In this section the results of the implementation of the probabilistic storm transposition approach to a semihypothetical example in the midwest are given. The purpose of this presentation is to partially investigate the robustness of the probabilistic approach to several simplifying assumptions, get a quantitative idea of the data available for estimation, and identify issues that need further study.

To achieve computational efficiency and flexibility and at the same time work with storms close to reality, I have transformed a set of extreme midwest storms to a set of idealized single-center storms which preserve the storm depth-area relationships at a specified duration. The single-center assumption is a convenient choice that permits the efficient computational processing of many storms, based on their DAD curves and an assumed storm shape. The DAD curves for a storm [World Meteorological Organization, 1969] result from lumping all areas of equal precipitation together so that the maximum average depth over a given area is always computed; the maximum DAD results from remapping the actual, often multicenter storm, to a single-center storm with geometrically similar isohyetal contours around the storm center. Therefore from the published DAD data one can obtain the spatial rainfall pattern of an idealized single-center storm but not the finer spatial structure. Because of the assumed simplified spatial storm structure, the results are interpreted on a comparative rather than an absolute basis.

The storm severity criterion E was arbitrarily chosen as

Criterion E

$$\{d[\Delta t = t_r, |A| = 10 \text{ mi}^2] \geq 13 \text{ in} \tag{15}$$

that is, $\mathcal{S}_E$ was composed of storms that had a maximum observed total depth over the whole storm duration exceeding 13 in (33 cm). Only storms that had their centers (defined as the stations with the maximum observed depth) within one of the nine midwestern states of North Dakota, South Dakota, Nebraska, Kansas, Minnesota, Iowa, Missouri, Wisconsin, and Illinois were used in the analysis. On the basis of the above two criteria and using the catalog of extreme storms [*U.S. Army Corps of Engineers*, 1948] for the 72-year period of 1882–1953, 18 storms were identified. These storms are listed in Table 1, together with their duration, maximum observed depth and location, maximum areal extent and associated average depth reported in the DAD tables [*U.S. Army Corps of Engineers*, 1948], and the month and year of occurrence. The geographical distribution of the storm centers is shown in Figure 5. The center

TABLE 1. Extreme Midwestern Storms Used

Storm Number	U.S. Army Corps of Engineers Number	Duration, hours	Maximum Total Depth, inches	Areal Extent, mi² (Associated Average Depth, inches)	Storm Center		Date
					Town	State	
1	MR 4-24	54	24	63,000 (3.5)	Boyden	IA	Sept. 1926
2	MR 4-5	20	13	20,000 (3.5)	Grant Township	NE	June 1940
3	MR 6-15	78	17.3	16,000 (4.6)	(41°52; 97°03)	NE	June 1944
4	MR 7-2A	78	19.4	45,000 (5.0)	Cole Camp	MO	Aug. 1946
5	MR 1-10	96	15.5	59,000 (4.4)	Woodburn	IA	Aug. 1903
6	MR 2-29	78	13.9	113,500 (3.9)	Grant City	MO	July 1922
7	MR 1-5	78	13.6	100,000 (3.9)	Primghar	IA	July 1900
8	MR 10-2	108	18.5	57,000 (6.0)	Council Grove	KS	July 1951
9	MR 8-20	120	17.6	50,000 (3.8)	near Holt	MO	June 1947
10	MR 1-9	168	16.8	136,000 (4.8)	Salina	KS	May 1903
11	MR 3-14	120	13.9	120,000 (5.0)	Pleasanton	KS	Oct. 1927
12	MR 4-2	96	13	30,000 (4.6)	Larrabee	IA	June 1891
13	UMV 1-11	198	13.2	50,000 (4.8)	Ironwood	MI	July 1909
14	UMV 2-9	120	13.1	57,100 (4.2)	Louisiana	MO	Aug. 1916
15	UMV 2-18	180	13	70,000 (6.2)	Booneville	MO	Sept. 1905
16	UMV 1-22	78	15	60,000 (4.7)	Hayward	WI	Aug. 1941
17	OR 4-8	90	15.4	70,000 (8.2)	Golconda	IL	Oct. 1910
18	SW 2-1	114	14	30,000 (4.2)	Neosha Falls	KS	Sept. 1926

One inch equals 2.54 cm. One square mile equals 2.6 km².

(maximum total depth equal to 13.2 in (33.5 cm)) of one of these storms (UMV 1-11) occurred outside the area of interest, but this storm had a second center (total depth equal to 12.1 inches (30.7 cm)) in northern Minnesota, so it was included in the analysis. The seasonal distribution of the 18 storms is shown in Figure 6, and the chronological distribution is shown in Figure 7.

For this illustrative example the maximum 24-hour patterns (Figure 8) were used; the analysis would have been similar if another duration had been selected. The spatial rainfall patterns were assumed to be described by a single-center elliptical pattern, following the relationship $\log \bar{d}(A) = a + b|A|^n$. This relationship was fitted to the maximum 24-hour depth-area curves of Figure 8 using a nonlinear weighted least squares approach (with weights inversely proportional to the area). Only the area of the storm enclosed within the contour of 3 inches (7.6 cm) was used in the fitting of the model. Since this area was much larger than the catchment areas, this assumption does not affect the extreme average catchment depths, but it is convenient since it screens out storm positions that would not deposit a significant depth over the catchment. Table 2 lists the parameter estimates and the mean square error as an indicator of the goodness of fit. Also, in the same table, the predicted and estimated average depths over areas of 10 mi² (26 km²), and 10,000 mi² (≈26,000 km²) are given for comparison purposes, as well as the estimated area enclosed by the contour of 3 inches (7.6 cm).

To complete the isohyetal description of these storms, the shape parameter c, which is the ratio of the minor to major axis of the elliptical pattern, is needed. An estimate of c for this example should be obtained from the maximum 24-hour isohyetal patterns which were not available. Thus c was estimated (Table 2) using the total storm isohyetal patterns (Figure 9) published by the *U.S. Army Corps of Engineers* [1948]. Previous investigators [e.g., *Huff*, 1967; *Hansen et al.*, 1982; *Foufoula-Georgiou and Wilson*, 1989] have shown that the shape parameter c does not seem to have significant regional variation and that the most frequent shape ratio for storms in the central plains is elliptical, with a major to minor axis ratio between 2 and 3, with 2 corresponding to storm areas between 5,000 and 50,000 mi² (≈13,000 and 130,000 km²) and 3 to larger areas. Since storms become much more complex and elongated (because of the movement dynamics) at the end of their duration, it is expected that the shape of the maximum 24-hour storms is much closer to elliptical, and

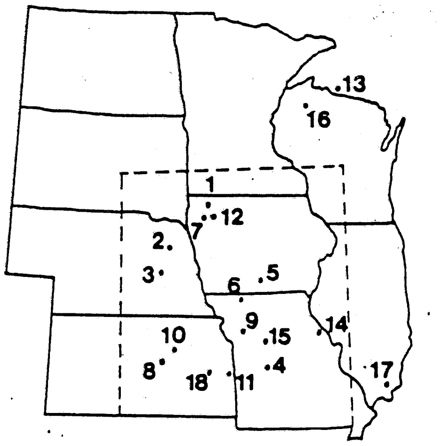

Fig. 5. Geographical distribution of the 18 storm centers.

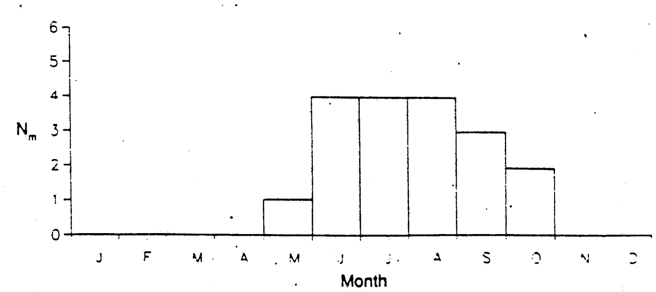

Fig. 6. Seasonal distribution of the analyzed extreme midwestern storms. N_m is number of storms in a month.

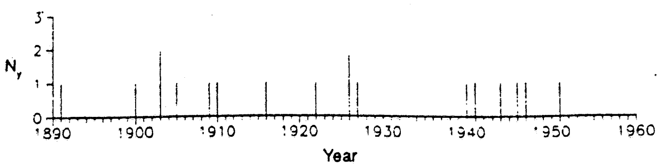

Fig. 7. Chronological distribution of the analyzed extreme midwestern storms. N_y is number of storms in a year.

the shape ratio is, in general, smaller than the one inferred from Figure 9. Because the shape parameter was estimated here in an arbitrary and approximate manner, a sensitivity analysis of the results on this approximation was performed, and the results are presented in section 8.

The geographical distribution of the storm centers (Figure 5) indicates that there is a tendency for extreme midwestern storms to be centered in the states of Iowa, Missouri, and the eastern parts of Nebraska and Kansas. None of the storms considered were centered in the states of North and South Dakota and Minnesota, although substantial parts of the last two states were covered by these storms. This preferred spatial location of extreme storms (which is also indicated in Figure 3) may be partially due to small or incomplete storm samples, althouugh it does also seem to be supported meteorologically; partial explanations for this pattern may be sought by connecting it to the observed movement characteristics of mesoscale convective complexes [e.g., *Maddox et al.*, 1982]. In this example, and for the relatively high storm severity criterion adopted, it is assumed that the storm position is independent of the storm characteristics within the study area, and therefore $f(\lambda_p|\lambda_s)$ in (4) is equal to $f(\lambda_p)$. In essence, $f(\lambda_p)$ is the first-order intensity of the spatial process of extreme storm occurrences within the transposition area. Since there is no physical reason suggesting the presence of clustering, it is hypothesized that the extreme storm occurrence process within the nine-state Midwestern region is an inhomogeneous spatial Poisson process. However, the spatially variable rate of this process is difficult to estimate from only 18 events. Thus, in this example, the storm transposition area A_{tr} has been arbitrarily defined as a smaller area within the major area (see broken line of Figure 9), such that the spatial occurrence of the storm centers within A_{tr} is a homogeneous Poisson process. The hypothesis of the homogeneous Poisson process within the area A_{tr} (which implicitly leads to equal probabilities of storm transposition within this area) was tested using well-known statistical tests [e.g., *Ripley*, 1981]. The area A_{tr} was estimated to 273,000 mi^2 ($\simeq$709,800 km^2). The sensitivity of the probability of exceedance estimates to the limits of the transposition area has not been investigated in this analysis. The study of *YAEC* [1984] considered several arbitrarily defined transposition areas and concluded that overall the results were not overly sensitive to the specification of these limits. This issue, however, needs further investigation given that several other simplifying assumptions were made by the YAEC study.

The temporal occurrence of extreme storms (Figure 7) is generally expected to follow a time homogeneous Poisson process. For this particular set of storms (defined by the high cutoff level of Criterion *E*) only 2 years experienced more than one extreme storm; these storms were far apart geographically and temporally, so that they could not have

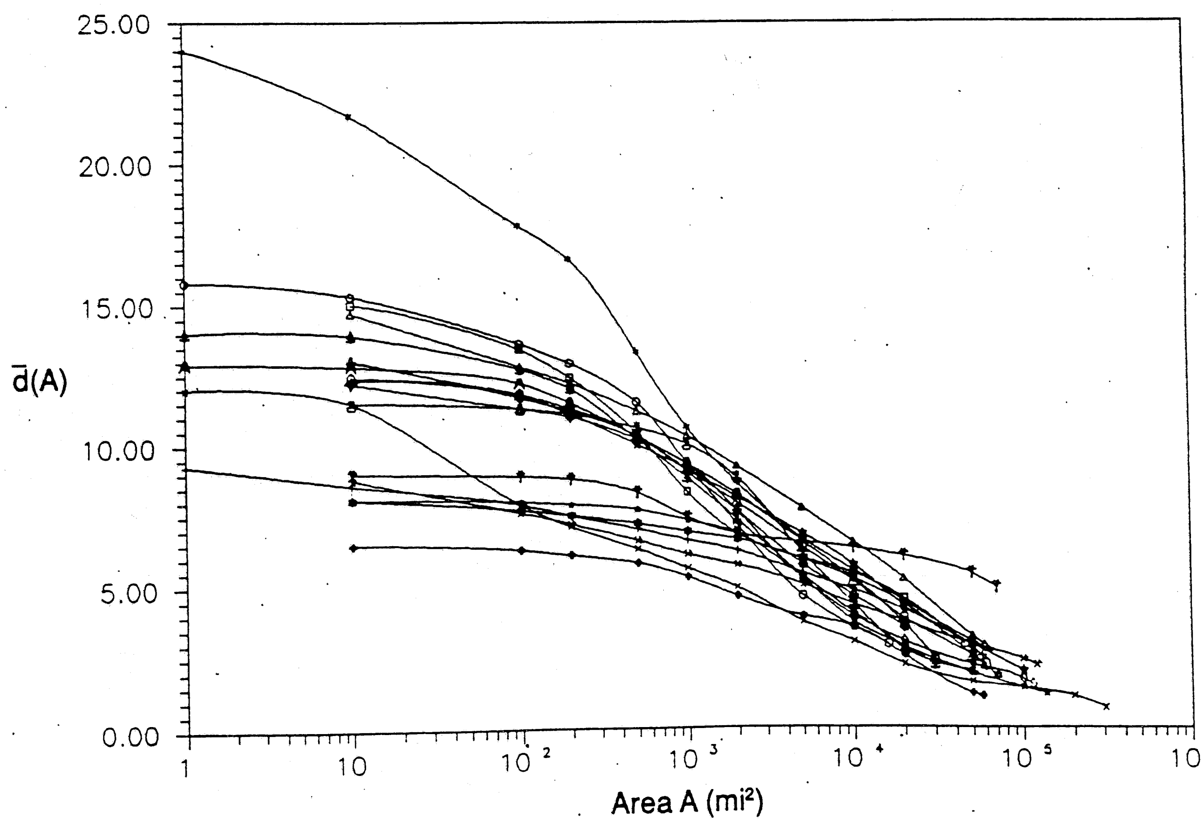

Fig. 8. Maximum 24-hour average-depth-area curves for the 18 analyzed storms. A is area (square miles); $\bar{d}(A)$ is average depth (inches) over area A.

TABLE 2. Estimates of the Parameters of the Spatial Rainfall Model*

Storm Number	$\hat{a}$	$\hat{b}$	$\hat{n}$	MSE	$\bar{d}(10)$ Measured	$\bar{d}(10)$ Predicted	$\bar{d}(10{,}000)$ Measured	$\bar{d}(10{,}000)$ Predicted	Area Enclosed in Contour of 3 Inches, mi²	Assumed $1/c$
1	1.41	−0.035	0.34	0.334	21.7	21.7	5.5	4.4	5,026	2
2	1.14	−0.010	0.42	0.040	13.0	13.0	4.6	4.6	7,733	2
3	1.21	−0.007	0.50	0.108	15.3	15.3	3.5	3.0	3,827	2
4	1.22	−0.016	0.39	0.533	15.0	15.0	5.4	3.9	6,484	3
5	1.24	−0.038	0.26	0.016	14.7	14.7	6.5	6.6	35,286	2.5
6	1.10	−0.003	0.57	0.035	12.2	12.2	3.9	3.8	4,288	3
7	1.10	−0.004	0.49	0.091	12.3	12.3	5.7	5.1	10,859	3
8	0.98	−0.023	0.28	0.002	8.6	8.6	4.8	4.8	22,524	2
9	2.16	−0.962	0.06	0.107	11.5	11.6	3.0	3.4	3,749	1
10	0.91	−0.0001	0.89	0.002	8.1	8.1	3.7	3.7	5,160	2
11	1.07	−0.081	0.18	0.005	8.8	8.8	4.3	4.4	22,908	2.5
12	1.12	−0.004	0.51	0.060	12.8	12.8	5.2	4.7	7,675	3
13	1.06	−0.0002	0.83	0.022	11.5	11.5	4.3	4.1	5,886	2.5
14	0.82	−0.002	0.56	0.030	6.5	6.5	3.5	3.1	3,983	3
15	0.92	−0.007	0.37	0.010	8.1	8.1	5.2	5.2	28,250	3
16	1.10	−0.003	0.53	0.067	12.4	12.4	5.1	4.6	8,637	2.5
17	0.96	−0.002	0.51	0.266	9.0	9.0	6.3	5.9	18,099	3
18	1.17	−0.007	0.48	0.075	13.9	13.9	4.0	3.5	5,088	2

One inch equals 2.54 cm. One square mile equals 2.6 km².

*Model is $\log \bar{d}(A) = a + bA^n$; mean square error (MSE) of the fitted model; measured and predicted average depths $\bar{d}(A)$, for $A = 10$ and 10,000 mi²; estimated area of the storm enclosed within the contour of 3 inches; and assumed elliptical shape parameter $1/c$ (c = ratio of minor to major axis).

influenced the same catchment. It was assumed therefore that the temporal occurrence of extreme storms (that could produce flooding of a catchment within a year) follows a Bernoulli process with success probability p_s, estimated as the number of extreme storms divided by the years of record. The estimation of the joint probability density function of the storm characteristics Λ_s from only 18 storms was deemed unreliable, and thus the integration over Λ_s in (4) was simply replaced by the summation over all 18 storms. Implicitly, this means that only the actual storms, and not other generated storms, were transposed over the catchment. It was also assumed that the catalog of extreme storms was complete during the period of 1882–1953 (72 years) with respect to storms with $d_0 \geq 13$ inches (33 cm).

On the basis of the above assumptions, an estimate of (4) becomes

$$\hat{I} = \sum_{j=1}^{N} \left\{ \iint\limits_{\substack{x\ y \\ (x,y) \in A_{\mathrm{eff},j}}} [1 - \hat{p}(\bar{d}_c \geq \bar{d} \mid x, y; (x, y) \in A_{\mathrm{eff},j})]\, dx\, dy \right\} (|A_{\mathrm{eff},j}|/|A_{\mathrm{tr}}|) \qquad (16)$$

where $N = 18$ is the number of extreme storms available for estimation, $A_{\mathrm{eff},j}$ is the effective catchment area relative to storm j, and (x,y) are the spatial coordinates of the storm center position. Thus (1) becomes

$$\hat{p}_a(\bar{d}_c \geq d) = \hat{p}_s \sum_{j=1}^{N} \left[\iint\limits_{\substack{x\ y \\ (x,y) \in A_{\mathrm{eff},j}}} \hat{p}(\bar{d}_c \geq d \mid x, y; (x, y) \in A_{\mathrm{eff},j})\, dx\, dy \right] (|A_{\mathrm{eff},j}|/|A_{\mathrm{tr}}|) \qquad (17)$$

By defining

$$p_j(\bar{d}_c \geq d) = \iint\limits_{\substack{x\ y \\ (x,y) \in A_{\mathrm{eff},j}}} p(\bar{d}_c \geq d \mid x, y; (x, y) \in A_{\mathrm{eff},j})\, dx\, dy \qquad (18)$$

(17) becomes

$$\hat{p}_a(\bar{d}_c \geq d) = \hat{p}_s \sum_{j=1}^{N} \hat{p}_j(\bar{d}_c \geq d)(|A_{\mathrm{eff},j}|/|A_{\mathrm{tr}}|) \qquad (19)$$

The probability $p_j(\bar{d}_c \geq d)$ is called here the conditional probability of exceedance and denotes the probability that the average depth over a catchment will exceed the value d given that storm j may occur anywhere within the effective area of the catchment. The effective areas are listed for all storms and the two hypothetical catchments, which are

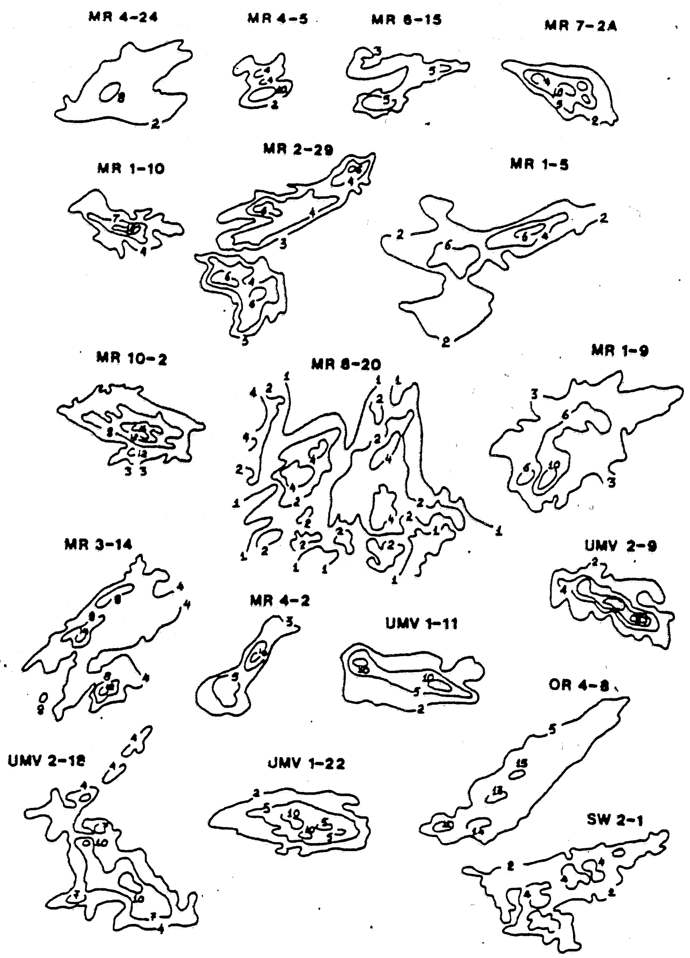

Fig. 9. Total storm isohyetal patterns of the analyzed extreme storms [from *U.S. Army Corps of Engineers*, 1948].

assumed circular and of areas 100 and 1000 mi^2 ($\approx$260 and 2600 km^2), in Table 3. In the same table the maximum average catchment depths ($\bar{d}_{c,\max}$) which result when each storm is centered over the catchments are given. Using a grid resolution of 1 mi^2 (2.6 km^2) (which after simulations was found adequate in terms of accuracy) and 1000 simulated storms with centers uniformly distributed within the effective storm area, the conditional probabilities $p_j(\bar{d}_c \geq d)$ were estimated from all the eighteen storms, $j = 1, 2, \ldots, 18$, and are shown in Figures 10 and 11 for the catchments of 100 mi^2 and 1000 mi^2 respectively. The estimates of the unconditional annual exceedance probability $p_a(\bar{d}_c \geq d)$ are shown in Figures 12 and 13.

Although the estimates may be pretty unreliable at this point, several observations can be made from these figures. There is a well-defined general tendency of the tails of the probability distributions to follow a smooth and well-behaved line (on a logarithmic-probability, arithmetic-depth scale) although not quite linear, as previous studies have suggested [e.g., *YAEC*, 1984]. The simplified storm transposition approach seems to overestimate the 100-year maximum 24-hour average depth inferred from the depth-duration-frequency curves of the central Iowa region. For example, the 100 mi^2 24-hour maximum average depth for central Iowa is approximately 6 in (15.2 cm) [*Hershfield*, 1961], while the value obtained from Figure 12 is approximately 8 in (20.3 cm). In other words, the storm transposition approach assigns to the 100 mi^2 maximum 24-hour average depth of 6 in a return period approximately equal to 60 years, while the intensity-duration frequency (IDF) curves assign to it a return period of 100 years. This is not surprising, given that the storm transposition approach has a degree of conservatism already built in, by considering the extreme storms over a much larger area as compared to the area over which the derivation of the IDF curves is based. It should be kept in mind, however, that the IDF approach estimates may well be within the confidence limits of the storm-transposition-approach estimates, especially in view of the several arbitrary assumptions made in this example.

It is worth noting that the 24-hour 100 mi^2 PMP for central Iowa is approximately 27 in (68.6 cm) [see *Schreiner and Riedel*, 1978]. This preliminary analysis cannot assess in any reliable way the return period of the PMP event. This event falls much outside the range of recorded extreme storms, and extrapolation cannot be made at this point in any reliable manner. Synthetic storms with less probable but more severe characteristics than those observed will be needed to assess the return period of PMP order events.

Although preliminary and oversimplified, the above analysis suggests that the storm transposition approach offers a promising method for estimation of the probability of exceedance of extreme precipitation depths. Further theoretical and empirical studies are needed to explore better estimation methods and quantify the sensitivity of the estimates to model assumptions and simplifications and also to parameter uncertainty. Such studies will provide insight on the degree of detail needed for reliable estimation and will screen out variables which do not considerably affect the estimates. In the next section the results are presented of a sensitivity analysis of the estimates on the specification of the storm and catchment shapes, which were arbitrarily assumed elliptical and circular, respectively, in the hypothetical example.

TABLE 3. Effective Area and Maximum Average Catchment Depth for all Storms Transposed Over Circular Catchments of Areas 100 and 1000 mi^2 ($\approx$260 and 2600 km^2)

	A_{eff}, mi^2		$\bar{d}_{c,\max}$, inches	
Storm Number	100 mi^2 Catchment	1000 mi^2 Catchment	100 mi^2 Catchment	1000 mi^2 Catchment
1	6,463	10,680	17.65	10.90
2	9,538	14,506	11.79	8.95
3	5,150	8,878	13.81	9.53
4	6,484	13,431	13.10	9.08
5	39,211	49,540	13.02	10.05
6	5,748	10,124	11.28	8.24
7	13,177	19,473	11.41	9.17
8	25,474	33,576	7.91	6.57
9	4,944	8,430	9.34	5.31
10	6,601	10,849	8.01	7.18
11	26,090	34,632	7.72	6.07
12	9,662	15,125	11.83	9.14
13	7,565	12,217	11.20	9.59
14	5,380	9,617	6.16	5.06
15	31,966	41,745	7.57	6.60
16	10,670	16,089	11.56	9.31
17	21,109	29,164	8.64	7.58
18	6,527	10,747	12.78	9.31

One square mile equals 2.6 km^2. One inch equals 2.54 cm.

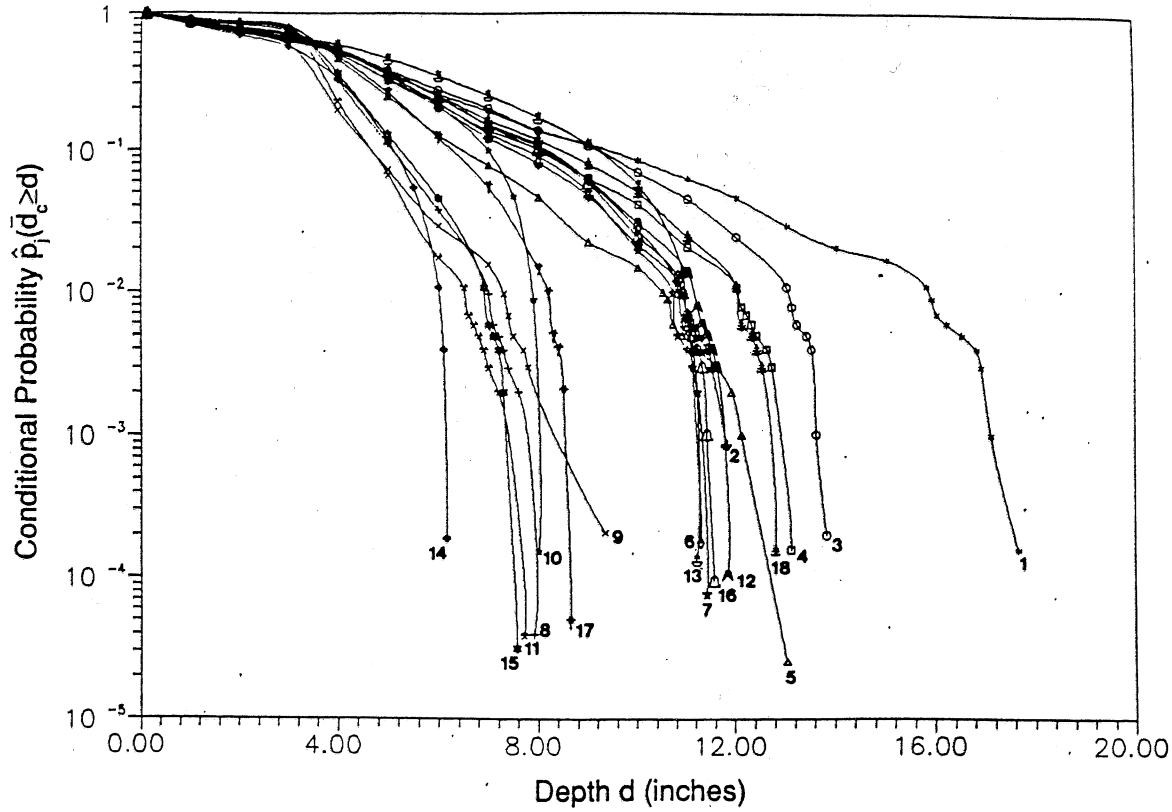

Fig. 10. Conditional probability of exceedance of the average catchment rainfall depth ($\bar{d}_c$) for all 18 storms (j = 1, 2, . . . , 18) transposed over a circular catchment of area 100 mi² (≈260 km²). See equation (18) for definition of the conditional probability.

8. Storm and Catchment Shape Idealization and Sensitivity of Results

Consider a circular catchment of area A_c and a stationary storm of area A_s. The shape of the storm is elliptical with minor to major axis equal to c. Assuming that the spatial distribution of the storm center is uniform and considering only storms which produce nonzero rainfall over the basin, we derive by simulation the first two moments of the fraction of the catchment covered by the storm (A_{sc}/A_c) as a function of the size of the catchment relative to the storm, that is, parameter $\alpha = (A_c/A_s)^{1/2}$. This analysis is similar to that of *Eagleson* [1984] and *Eagleson and Wang* [1985], where, however, only the mean and variance of the wetted area were derived analytically for the special case of circular storms and circular catchments. The grid size used for estimation was 1 mi × 1 mi (1.6 km × 1.6 km), and 1000 storms were generated, with their centers uniformly distributed within the effective area of the catchment. The grid resolution of 1 mi² was tested thoroughly to insure that it provided sufficient numerical accuracy for the catchment and storm sizes used. Figures 14 and 15 show the plots of the expected value and standard deviation of (A_{sc}/A_c) as a function of $(A_c/A_s)^{1/2}$ for a circular storm and elliptical storm, with $1/c$ = 1.5, 2.0, and 3.0. Although differences are observed, these are not very critical. The differences could be more pronounced if a spatially varying (and not constant) depth was used for the storm, as is illustrated below.

Figure 16 shows the conditional probability density function of the average depth over a circular catchment of area 1000 mi² (2600 km²) resulting from two elliptical storms with shape parameter $1/c$ = 2.5 and 3.0 and isohyetal pattern given by equation (13), with parameters a = 1.41, b = −0.035, and n = 0.34. (These parameters correspond to storm 1 of Table 1.) The areal extent of the storm, defined as the area enclosed within the contour of 3 in (7.6 cm), was 5026 mi² (13,068 km²). Differences occur for the events of interest (return periods greater than 50 or 100 years). These differences are even more pronounced if a circular storm (c = 1) were used. From the above analysis it is concluded that misspecification of the storm shape may result in biased estimates of the probability of extreme depths over a catchment. Whenever possible, the shape parameters should be estimated from the actual storm patterns at the duration of interest and not from the published total storm patterns, as done in the illustrative example. Also, the results of previous studies where circular storms have been assumed may be considerably biased.

In the example of the previous section the 18 idealized storms have been transposed over two circular hypothetical catchments of 100 mi² and 1000 mi² centered in Iowa. The circular catchment shape was chosen for convenience and also to avoid the need for specifying a distribution of the storm orientations, since for a circular catchment any storm orientation will produce the same probability density function of the average catchment depth. Further analysis of the storm transposition methodology should take into account the actual storm orientation and/or the probability distribution of the storm orientations given that, at least in the midwest, extreme storms seem to follow a preferred orientation and movement pattern [e.g., *Huff and Semonin*, 1960; *Foufoula-Georgiou and Wilson*, 1989].

To get an insight of how the results would apply to a

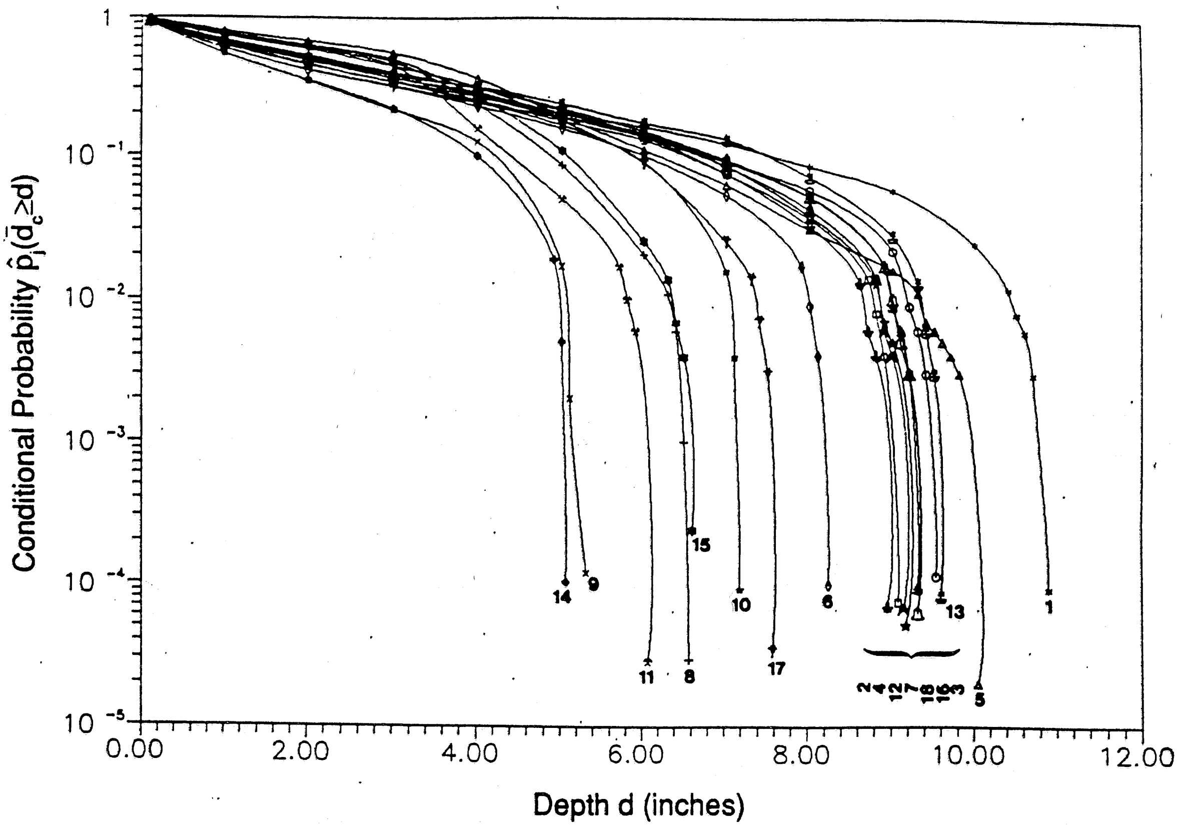

Fig. 11. Conditional probability of exceedance of the average catchment rainfall depth ($\bar{d}_c$) for all 18 storms ($j = 1, 2, \ldots, 18$) transposed over a circular catchment of area 1000 mi^2 ($\simeq$2600 km^2). See equation (18) for definition of the conditional probability.

catchment of the same area but different shape, some simulation experiments involving four different catchment shapes (circular, rectangular, triangular, and elliptical) and circular storms of constant intensity were performed. The relative dimensions of the catchments are shown in Figure 17. Rectangular catchments for $h_1/h_2 = 1, 2$, and 4 (cases R_1, R_2, and R_3); triangular catchments for $h_1/h_2 = 1$ and 1.5 (cases T_1 and T_2); and elliptical catchments for $1/c = 2$ and 3 (cases E_1 and E_2) were used. Figures 18 and 19 show that the moments of the wetted catchment area are relatively sensitive to the shape of the catchment. This points to the need of incorporating the storm/catchment interactions in any hydrologic analysis involving runoff production. Previous studies [e.g., *Alexander*, 1963; *YAEC*, 1984] have only approximately accounted for these interactions and have downplayed their effect on the variability of the average catchment depth and thus on the variability of the produced runoff.

9. Summary and Conclusions

The concept of probable maximum precipitation was introduced 50 years ago, and over the years it has undergone

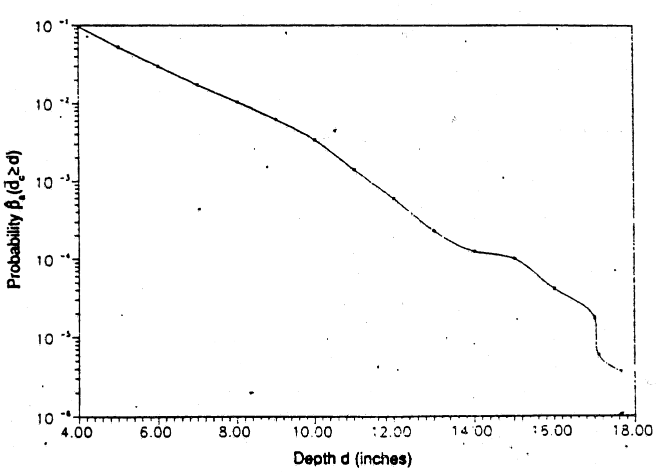

Fig. 12. Annual exceedance probability of the average catchment rainfall depth $\bar{d}_c$. Circular catchment of area 100 mi^2 ($\simeq$260 km^2).

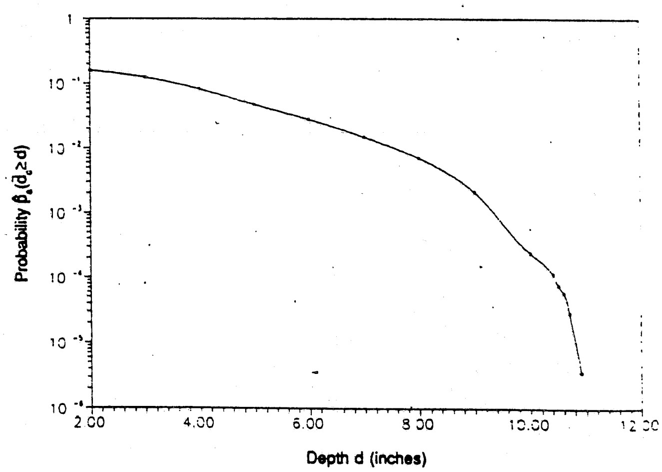

Fig. 13. Annual exceedance probability of the average catchment rainfall depth $\bar{d}_c$. Circular catchment of area 1000 mi^2 ($\simeq$2600 km^2).

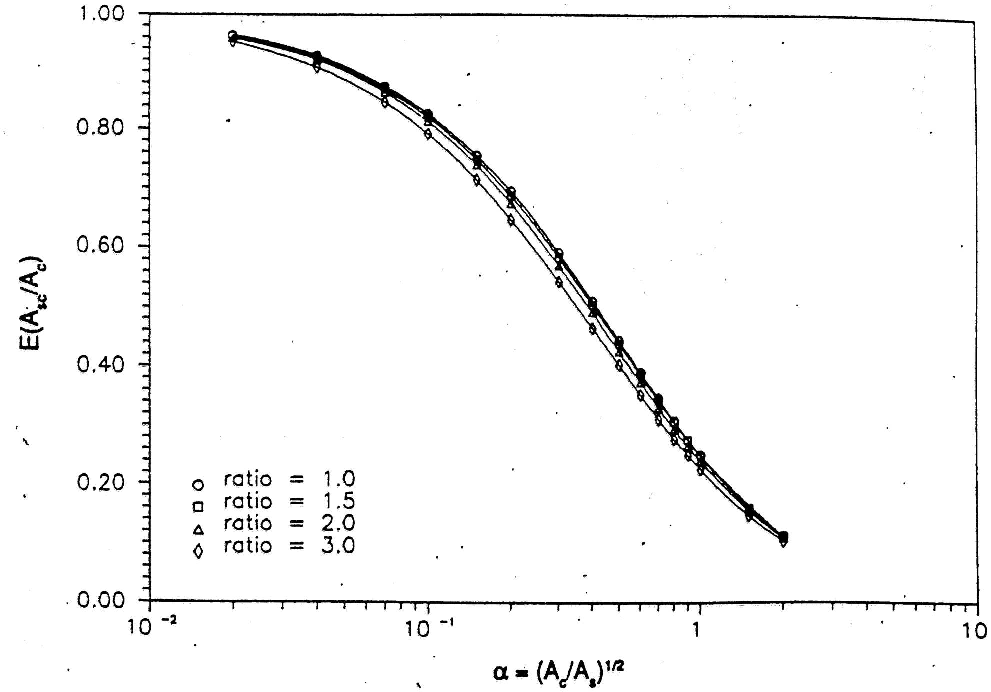

Fig. 14. Expected value of the fraction of a circular catchment covered by an elliptical storm as a function of the size of the catchment relative to the storm. Elliptical storms with major to minor axis equal to 1.0., 1.5., 2.0, and 3.0 are compared.

several modifications, improvements, and generalizations [*Hansen*, 1986*a*, *b*]. Recently, there has been an initiative for a movement away from the PMP-based methods to risk-based approaches for engineering design (see, for example, *Stedinger and Grygier* [1985], *Dawdy and Lettenmaier* [1987]; *NRC* [1988], and *Wallis* [1988]). A recent study by the *Interagency Advisory Committee on Water Data* [1986], after reviewing 230 papers on methods dealing with risk assessment of extreme floods, recommended that current design practices should be continued because no procedure proposed to date is capable of assigning an exceedance probability to the PMF or to near-PMF floods in a reliable, consistent, and credible manner. Although the motivation of the work reported herein is risk-based engineering design, the question posed is not What is the exceedance probability of PMP/PMF? (One could even dispute the very concept of

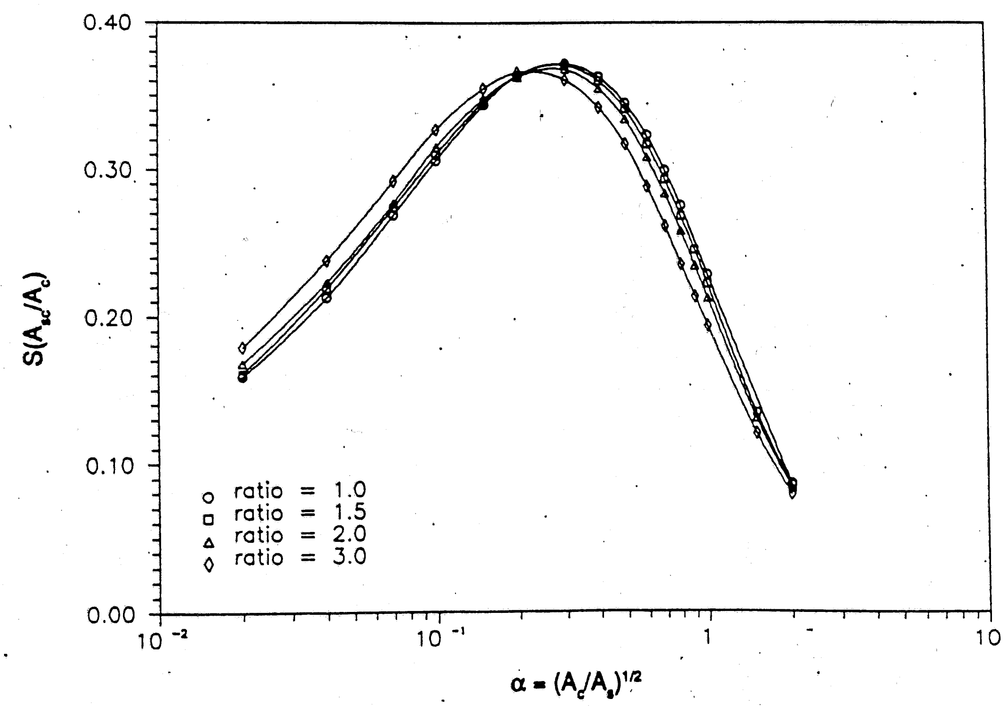

Fig. 15. Standard deviation of the fraction of a circular catchment covered by an elliptical storm as a function of the size of the catchment relative to the storm. Elliptical storms with major to minor axis equal to 1.0., 1.5., 2.0, and 3.0 are compared.

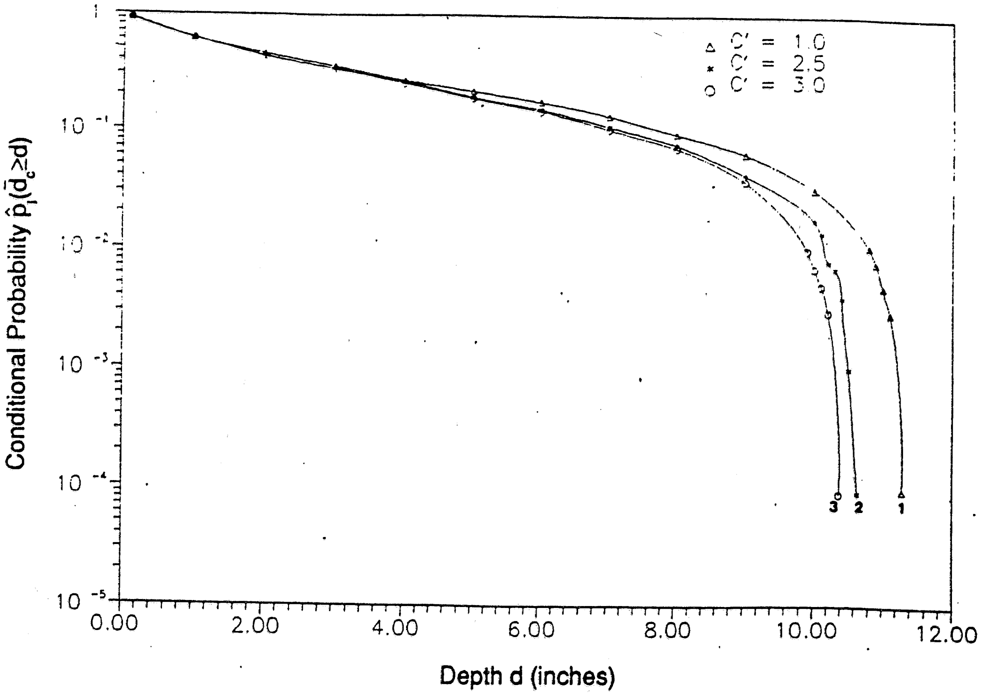

Fig. 16. Conditional probability of exceedance of the average catchment rainfall depth from an elliptical storm with isohyetal pattern described by $\log \bar{d}(A) = a + b|A|^n$ with $a = 1.41$, $b = -0.035$, $n = 0.34$ and shape parameter $C' = 1/c = 1.0, 2.5$, and 3.0 ($j = 1, 2, 3$). Storm transposed over a circular catchment of area 1000 mi^2 ($\approx$2600 km^2).

PMP/PMF as a design criterion, but this is outside the scope of the present paper.) Instead, the question posed is How can one use in a systematic manner storm, basin, and flood data to estimate the upper tail of the probability distribution of precipitation depths and resulting floods? The analysis reported here is a first attempt toward thoroughly exploring the first part of such a procedure, that is, estimation of the upper tail of precipitation depths.

In this paper the probabilistic storm transposition approach was formulated, and the several conceptual and estimation difficulties arising in such an approach were identified. Also, a simplified example of its implementation was presented using two hypothetical catchments in Iowa and 18 extreme midwestern storms. The tail of the unconditional annual exceedance probability of the average catchment depth was found to follow a smooth and well-behaved curve, supporting the idea of researching methodologies to extrapolate it to lower probabilities. The analysis shows that the storm transposition approach may yield more conservative 100-year return period events, as compared to those obtained by the regional depth-duration-frequency curves. (A conditional estimation procedure could be used so that consistency of the 100-year estimate is guaranteed.) Our results do not permit even a preliminary assessment of the exceedance probability of the PMP events since these fall far beyond the maximum depth resulting from the analyzed storms.

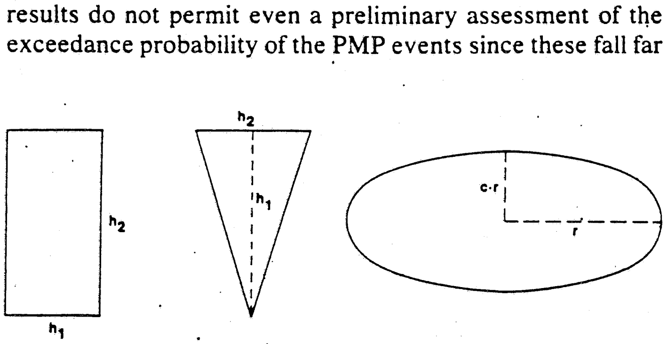

Fig. 17. The three catchment shapes used in Figures 18 and 19.

There are many critical issues that remain to be investigated before this approach can be used for extreme storm probability assessment. Some of these issues are (1) investigation of procedures for objectively defining the limits of the transposition area and estimating the inhomogeneous storm transposition probabilities $f(\boldsymbol{\lambda}_p|\boldsymbol{\lambda}_s)$, (2) investigation of more detailed extreme rainfall models and estimation of the probability distribution of the corresponding storm descriptors $f(\boldsymbol{\lambda}_s)$, (3) investigation of the incompleteness of the catalog of extreme storms and its effects on the estimation, and (4) investigation of the effect of the inaccuracy of the observed rainfall depths and especially the maximum observed depth, which is usually used for storm center positioning and possible ranking of storms, on the estimation of the spatial storm pattern and therefore on the probability distribution of the average depth over the catchment. Furthermore, research is needed to assess the reliability of the estimates, that is, estimate standard errors of the estimates.

Several studies [e.g., *Stedinger and Grygier*, 1985; *Karlsson and Haimes*, 1988] have demonstrated that the decision-making process is sensitive to the assumed return period of extreme design events such as PMF. This sensitivity is even more pronounced if one considers approaches based on the low-probability/high-consequences conditional expected risk [*Karlsson and Haimes*, 1988]. From the practical engineering standpoint one would only need estimate the return period of design events with an accuracy which falls within the range of return periods that would not significantly affect the design or decision-making process. Of course, this range of values depends on the particular problem. This is a fact that should be recognized when the problem of reliability of estimates of exceedance probabilities of extreme precipitation depths and floods is considered.

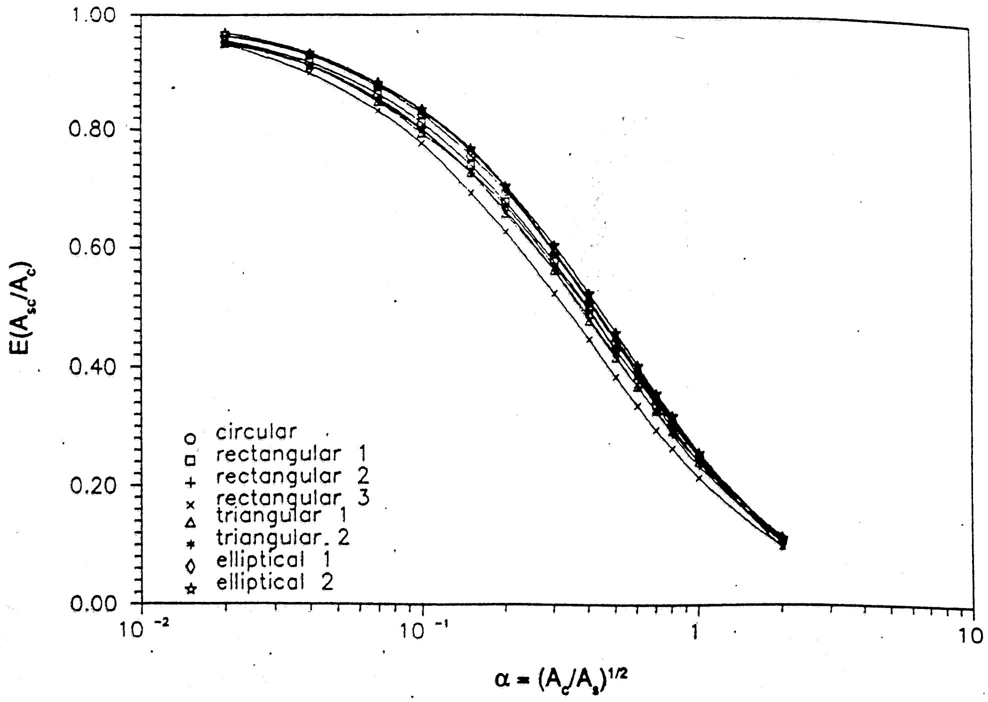

Fig. 18. Expected value of the fraction of the catchment covered by a circular storm. The shape of the catchment is circular, rectangular, triangular, and elliptical (as described in Figure 17). (R_1, $h_1/h_2 = 1$; R_2, $h_1/h_2 = 2$; R_3, $h_1/h_2 = 4$; T_1, $h_1/h_2 = 1$; T_2, $h_1/h_2 = 1.5$; E_1, $c = 1/2$; E_2, $c = 1/3$).

NOTATION

Λ_p random vector of storm center position, i.e., spatial coordinates of storm center.

Λ_s random vector of storm characteristics.

$\bar{\Delta t}$ the time interval over which the cumulative depth (over a specified area) is maximum as compared to all other depths accumulated over time periods of equal length Δt.

$\bar{A}$ the area of the storm over which the average depth (over a specified time period) is maximum as compared to the average depth over any other area of equal extent.

$\bar{d}_c = \bar{d}_c(\Delta t)$ average rainfall depth deposited over the catchment during a time interval Δt.

$d[\Delta t, A]$ storm depth accumulated over a period Δt and averaged over an area A.

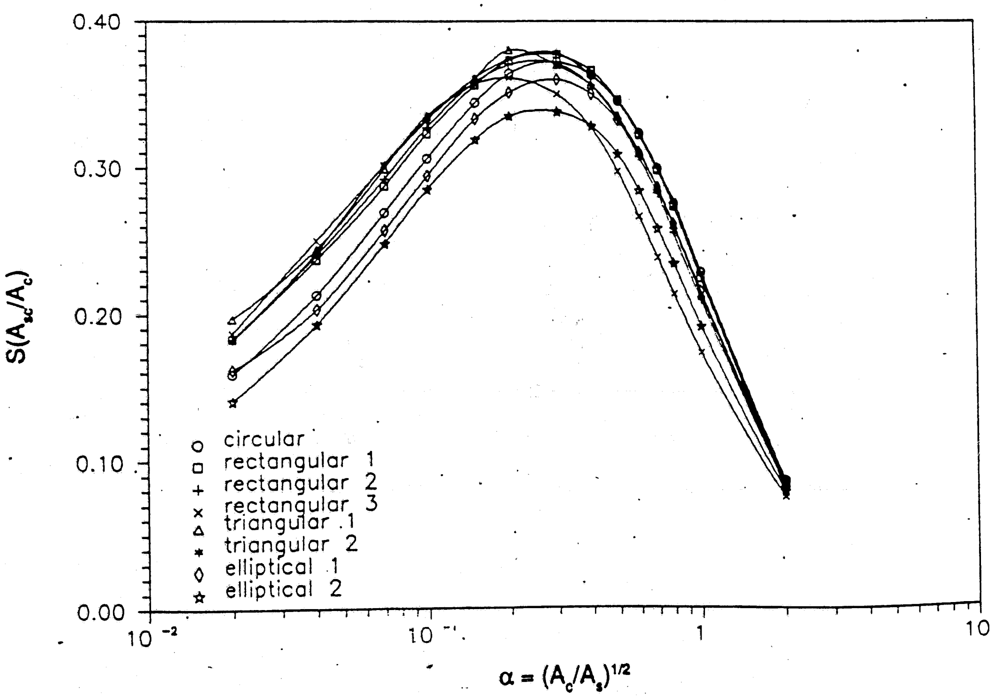

Fig. 19. Standard deviation of the fraction of the catchment covered by a circular storm. The shape of the catchment is circular, rectangular, triangular, and elliptical (as described in Figure 17). (R_1, $h_1/h_2 = 1$; R_2, $h_1/h_2 = 2$; R_3, $h_1/h_2 = 4$; T_1, $h_1/h_2 = 1$; T_2, $h_1/h_2 = 1.5$; E_1, $c = 1/2$; E_2, $c = 1/3$).

$d(\Delta t, x, y)$ storm depth accumulated during a period Δt over the point of spatial coordinates (x, y).
$\bar{d}(A)$ average storm depth over an area A.
$d(A)$ value of the depth along the isohyet enclosing an area A.
A_{tr} storm transposition area.
A_c catchment area.
A_s storm area.
A_{sc} area of the catchment covered by a storm.
$A_{eff,j}$ effective catchment area relative to storm j.
ϕ storm orientation.
c minor to major axis in an elliptical storm.
$Z(t)$ number of extreme storms in $(0,t)$ years.
$p_j(\bar{d}_c \geq d)$ conditional probability of exceedance equal to probability that the average catchment depth will exceed the value d, given that storm j may occur anywhere within the effective area of the catchment.
$p_a(\bar{d}_c \geq d)$ unconditional annual exceedance probability of the average catchment depth.

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