6 An Exponential Langevin-type Model for Rainfall Exhibiting Spatial and Temporal Scaling

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Abstract. Models for simulating the space-time structure of rainfall over a range of scales remain a valuable tool for hydrologic and geomorphologic studies. A desired attribute of these models is the preservation of statistical properties of observed precipitation fields, among which are well documented spatial and temporal scaling. In this paper we propose a class of stochastic space-time rainfall models based on exponentiation of evolving nonlinear Langevin-type surfaces. The proposed models have the potential of reproducing (a) the spatial and temporal scaling structure of observed rainfall fields, (b) the spatial and temporal intermittency in rainfall over a range of scales, and (c) possible nonstationarity in space and time.

1 Introduction

Precipitation varies simultaneously over space and time. Although several studies have concentrated on the scaling structure of spatial rainfall fields and separately on the scaling structure of temporal rainfall (See Lovejoy and Schertzer 1990; Gupta and Waymire 1993; Kumar and Foufoula-Georgiou 1993, Carsteanu and Foufoula-Georgiou 1996; among others; see also the review papers of Foufoula-Georgiou and Krajewski 1995; Foufoula-Georgiou 1998), it is obvious that space and time are not independent of each other and should be considered simultaneously. Towards this direction, the studies of Over and Gupta (1996); Marsan, Schertzer, and Lovejoy (1996); and Venugopal, Foufoula-Georgiou, and Sapozhnikov (1999) are worth noting. Over and Gupta (1996) proposed a space-time model based on a multiplicative cascade in space and a stochastic evolution in time. At each time step, the spatial field is generated by using cascade generator weights that are obtained via a Markov process from the weights at the previous time step. Marsan et al. (1996) proposed a space-time multifractal model which uses a three-dimensional anisotropic multiplicative cascade. They used a continuous cascade generated from a white Levy noise fractionally integrated at the smallest scale present. Venugopal et al. (1999) reported evidence of dynamic scaling in normalized rainfall fluctuations and proposed a space-time stochastic downscaling model which can also be used for simulation purposes. An alternative model was recently proposed by Kundu and Bell (2006) based on white-noise-driven fractional diffusion and it was shown to exhibit dynamic scaling.

In this paper, we explore a different class of models for space-time rainfall based on continuum growth equations and specifically the nonequilibrium models described by the Langevin-type equation introduced by Kardar, Parisi and Zhang (1986). These models have been extensively studied in recent years, because they exhibit interesting scaling properties, are simple and still are able of capturing important features of physical processes such as depositional surface growth (e.g., see Edwards and Wilkinson 1982; Meakin, Ramanlal, Sander, and Ball 1986; Family 1986; Lam and Family 1991), fluid displacement in porous media (e.g., Rubio, Edwards, Dougherty, and Gollub 1989; Kahanda Zou, Farrel, and Wong 1992; He, Kahanda, and Wong 1992), the growth of bacterial colonies (e.g., Vicsek, Cserzo, and Horváth 1990), diffusion-driven evolution of interfaces (Sapozhnikov and Goldiner 1991, 1992), turbulence (e.g, Kardar et al. 1986; Schwartz and Edwards 1992; see also references in Bohr, Jensen, Paladin, and Vulpiano 1998), landscape evolution (e.g., Czirok, Somfai, and Vicsek 1993; Sornette and Zhang 1993; Stomfai and Sander 1997; Banavar, Colaiori, Flammini, Saritan, and Rinaldo 2001; see also Passalacqua, Porté-Agel, Foufoula-Georgiou, and Paola 2006), and others. In this article we use Langevin-type surfaces to build a space-time model for rainfall and demonstrate that it reproduces observed space-time scaling properties.

In the next section, a brief review of the surface growth models and especially the KPZ (Kardar et al. 1986) model and its space and time scaling characteristics is given. In section 3, an adaptation of the KPZ model for space-time rainfall simulation is proposed and shown to reproduce certain empirical scaling characteristics found in rainfall. Finally conclusions and directions for further research are discussed in section 4.

2 Surface Growth Models

The geometry and evolution of surfaces formed by stochastic processes causing roughening and smoothing, e.g., deposition, dendritic growth and diffusion-driven evolution of interfaces, has been studied theoretically and by computer simulation (see, e.g. Edwards and Wilkinson 1982; Kardar et al. 1986; Meakin et al. 1986; Family 1986; Lam and Family 1991; Sapozhnikov and Goldiner 1992; and Barabasi and Stanley 1995). These surfaces have been shown to exhibit double scaling, both in space and in time. Let us denote by $h(\mathbf{x},t)$ the height of the surface at location $\mathbf{x}$ and at time t, by $\overline{h(\mathbf{x},t)}$ the average height of the surface over the whole system of size L, and by $\sigma(L,t)$ the surface width, which characterizes the roughness of the surface, defined as the root mean square (RMS) fluctuation in height,

$$\sigma(L,t) \equiv < [h(\mathbf{x},t) - \overline{h(\mathbf{x},t)}]^2 >_x \qquad (1)$$

where $< >_x$ denotes spatial (over $\mathbf{x}$) average over the size of the system L. For many growth surfaces it has been found that initially the width increases as a power of time

$$\sigma(L,t) \sim t^{\beta}, \qquad t \ll t_{\times} \qquad (2)$$

where β is called the growth exponent and characterizes the time-dependent dynamics of the roughening process. The power law increase of width continues up

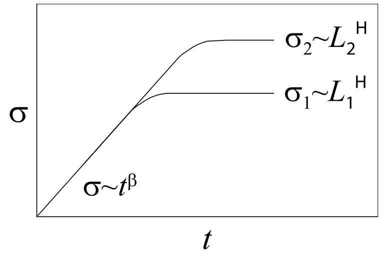

Fig. 6.1. Schematic of the log-log dependence of the $h(x, y, t)$ surface width σ on time, for two different linear sizes of the lattice L. Each of the two lines saturates at $\sigma \sim L^H$.

to a time $t_\times$ (called the cross-over time) at which the width reaches a saturation value σ_{sat} which depends on the size of the system. As the size L of the system increases, so does σ_{sat} and the dependence follows a power law

$$\sigma_{sat}(L) \sim L^H, \qquad t \gg t_\times \tag{3}$$

The exponent H is called the roughness exponent and characterizes the roughness of the saturated surface.
The cross-over time $t_\times$ at which the surface crosses from the behavior of (2) to that of (3) depends on the system size

$$t_\times \sim L^z \tag{4}$$

where z is called the dynamic exponent and can be shown to be

$$z = H/\beta \tag{5}$$

The above equations can be combined into one double-scaling relation (Family and Vicsek 1985; or Barabasi and Stanley 1995)

$$\sigma(L, t) = L^H f(t/L^z) \tag{6}$$

where $f(\xi) \sim \xi^\beta$ as $\xi \to 0$ (giving $\sigma \sim t^\beta$) and $f(\xi) \to const$ as $\xi \to \infty$ (giving $\sigma \sim L^H$) (see Figure 6.1).

It has also been shown that the H exponent is the Hurst exponent of the $h(\mathbf{x}, t)$ surface at any time t_0 after σ stabilizes, and thus describes the geometry of this surface at any given instance of time t_0. Furthermore, as discussed later, β is equal to the Hurst exponent H_t of the surface evolution at any fixed point $\mathbf{x}_0$, i.e., of $h(\mathbf{x}_0, t)$. Based on the scaling exponents β and H, such surfaces have been found to fall into distinct classes of universality, depending on the character of the processes that give rise to the surface evolution (e.g., see Liu and Plischke 1988; Krug 1987; Barabasi and Stanley 1995).

To connect the shape and evolution of growing surfaces to the forming processes, Edwards and Wilkinson (1982) suggested a linear equation of Langevin type and showed that it yields the spatial and temporal scaling of the surface. This work was later extended by Kardar et al. (1986). They proposed the following nonlinear equation of Langevin type for surface growth (often referred to as the KPZ equation)

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = \nu\nabla^2 h(\mathbf{x},t) + (\lambda/2)(\nabla h(\mathbf{x},t))^2 + \eta(\mathbf{x},t) \tag{7}$$

where $h(\mathbf{x},t)$ measures the height of the surface relative to the average height and ν and λ are model parameters. The first term in the right-hand side describes the surface smoothing, the second (nonlinear) term is caused by slope dependence of the surface growth rate, and the third term is white noise describing roughening of the surface by a random process. In case $\lambda = 0$ this relation is reduced to the linear equation of Edwards and Wilkinson (1982), which gives $H = 1/2$ and $\beta = 1/4$ for 1-dimensional surfaces $h(x,t)$. Kardar et al. (1986) solved equation (7) analytically for 1-dimensional surfaces $h(x,t)$ and showed that if $\lambda \neq 0$ the exponents in (6) are $H = 1/2$, and $\beta = 1/3$.

To our knowledge equation (7) has not been solved analytically for two-dimensional surfaces $h(x,y,t)$. However, theoretical consideration of equation (7) (Halpin-Healy 1989) and computer simulation (Meakin et al. 1986; Hirsh and Wolf 1986; Liu and Plischke 1988; see also summary Table in Barabasi and Stanley 1995 p.83) suggest that in this case H is estimated to be in the range of 0.36 to 0.385, β in the range of 0.23 to 0.24 and consequently z is approximately 1.6 (Note that these values agree with the values obtained from the equations $H = 2/(d+3)$ and $\beta = 1/(d+2)$ proposed by Kim and Kosterlitz 1989). In subsequent works of Medina, Hwa, Kardar, and Zhang (1989) and Lam and Family (1991) correlated noise was introduced instead of white noise in equation (7). In this case, the scaling exponents were shown to depend on the noise correlation structure. Moreover, Meakin et al. (1986), Krug (1987), and Medina et al. (1989) provided arguments that the nonlinear $(\lambda \neq 0)$ version of Langevin equation (7) yields the scaling exponents H and β connected by the simple relation

$$H + H/\beta = 2 \tag{8}$$

for both correlated and uncorrelated noise and in space of any dimension. Equation (8) provides a relation between the two scaling exponents and has been confirmed by the renormalization group calculation (see Barabasi and Stanley, Chapter 7 1995).

The linear version of the Langevin-type model in two dimensions exhibits no spatial scaling. As shown by Edwards and Wilkinson (1982), the surface produced exhibits logarithmic divergence. Thus this type of model would not be appropriate for rainfall modeling. Therefore we have adopted the nonlinear Langevin-type equation (7), which is also referred to as the KPZ equation, as the building block for the development of a space-time rainfall model.

3 Rainfall Model

Several models exist for simulating the nonlinear Langevin-type equation (7), such as the ballistic deposition model (Family and Vicsek 1985), the Eden growth model (Hirsh and Wolf 1986), or the single-step model of surface growth (Meakin et al. 1986; Liu and Plischke 1988) [The reader is referred to Barabasi and Stanley 1995 for details and other models]. Here, the single step model was chosen because of its simplicity and computational efficiency.

In the single-step model, first a "chessboard" initial surface is produced on a square lattice as

$$h(x,y,0) = 0.5[1+(-1)^{x+y}] \tag{9}$$

so the heights of neighboring columns differ by ± 1. Then sites of the lattice are chosen randomly, with equal probability, and if the chosen site is local minimum its height is increased by 2. One can notice that under this deposition procedure the heights of neighboring columns at any time can differ only by ± 1. Meakin et al. (1986) found that this model shows double scaling, with exponents $H = 0.36$, $z = 1.64$ (and thus $\beta = H/z = 0.22$). Liu and Plischke (1988) obtained similar results ($H = 0.375,\ z = 1.625$, and consequently $\beta = 0.23$).

As explained before, the surface average width σ stops increasing after long enough evolution time (indeed, in equation (6) $f(\xi) \to const$, e.g., σ stops depending on t, as $\xi \to \infty$). Usually the Langevin-type models of surface growth are studied at nonstationary stages, when the average width σ of the interface still grows. To simulate rainfall however, we first let the surface evolve enough such that its average width σ reaches the asymptotic value, and then follow the evolution of the surface.

It is an interesting result (e.g., see chapter 2 of Barabasi and Stanley 1995), verified also by us via computer simulation of surfaces corresponding to the linear and the nonlinear versions of the Langevin equation, that after the surface width σ reaches its limiting value, the β exponent, instead of describing the surface growth as in (6), determines how the height of the surface $h(x_0, y_0, t)$ at any point (x_0, y_0), fluctuates with time, e.g., that β is actually the Hurst exponent H_t of the $h(x_0, y_0, t)$ process. We will make use of this result for modeling space-time rainfall by the evolution of the stationary surfaces produced by the KPZ equation upon saturation.

3.1 Stationary rainfall

In the original single-step model, the surface average height increases linearly with time (it does not increase if one introduces evaporation with the same probability as deposition, but this, as found by Liu and Plischke (1988) turns the model into a linear one, with no spatial scaling in two dimensions). The linear growth of the surface average height would lead to a permanent growth in rainfall intensity with time, which is unrealistic. Therefore, we modified the model to preserve the average surface height, by shifting the surface back as a whole, after each deposition cycle. One could say instead that we considered the surface evolution in the coordinate system bound to the average height of the $h(x,y,t)$ surface. This approach is also consistent with the Langevin equation (7) where $h(\mathbf{x},t)$ measures the height of the surface relative to its average height. Each cycle included N^2 attempts of random deposition, equal to the number of sites in the lattice. Although the average height of the surface and (after the initial growth) the average width σ remained constant, the surface evolved continuously during the simulation. For example, Fig. 6.2(a) displays two cross-sections of the $h(x,y,t)$ surface along the $x-$direction ($y = y_0$), at two instances of time t_1 and t_2 , i.e., it shows $h(x,y_0,t_1)$ and $h(x,y_0,t_2)$. The differences seen between the two cross sections are the result of the surface evolution. Alternatively, if the surface evolution is followed over time at one specific point (x_0,y_0) , a temporal sequence $h(x_0,y_0,t)$, such as that shown in Fig. 6.3(a) is obtained.

The $h(x,y,t)$ surface was then used to simulate a rainfall field $I(x,y,t)$ as

$$I(x,y,t) = exp(Ah(x,y,t) + B) \tag{10}$$

Here A accommodates arbitrariness of units for $h(x,y,t)$ and B (which amounts to a pre-exponent) arbitrariness of the $I(x,y,t)$ units. It can be shown that the A coefficient is related to the temporal and the spatial scale of the process (e.g., see Barabasi and Stanley 1995). Indeed, if the $h(x,y,t)$ surface exhibits scaling in space and in time, with the exponents H (same for the x and the y - directions) and H_t , respectively, then

$$Ah(x,y,t) = h(A^{1/H}x, A^{1/H}y, A^{1/H_t}t) \tag{11}$$

Thus multiplying the surface height by A is equivalent to its space and time rescaling. Figures 6.2(b,c) show the temporal evolution of two cross-sections of the simulated rainfall field $I(x,y,t)$, at two different time instances. They have been obtained from exponentiation of the $h(x,y,t)$ cross sections shown in Fig. 6.2a, with two different values of A . Similarly, Figs. 6.3(b,c) show two temporal sequences of

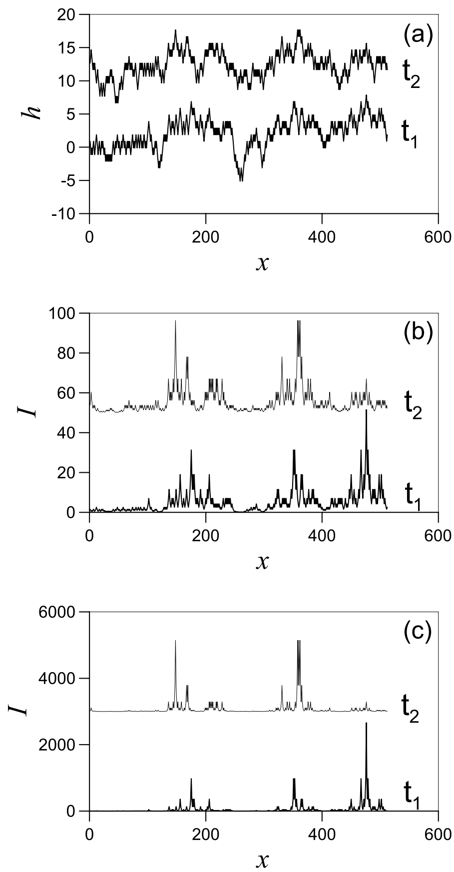

Fig. 6.2. (a) Cross-sections of the evolving surface $h(x, y, t)$ at two different time instances t_1 and t_2, 20 iteration steps apart; (b) Cross sections of the simulated rainfall field $I(x, y, t)$ produced by exponentiation of the $h(x, y, t)$ surfaces whose cross sections are shown in (a) and for $A = 0.5$, $B = 0$; and (c) same as (b) but with $A = 1$, $B = 0$. To avoid overlapping, the cross sections at time $t = t_2$ in (a), (b) and (c) have been shifted upwards by 10, 50 and 3000 units, respectively.

simulated rainfall intensity at a point, for different A and B values. These sequences look similar to observed temporal rainfall sequences shown in Figs. 6.3(d,e) for two Iowa storms in November 1, 1990 (see Georgakakos, Carsteanu, Sturdevant, and Cramer 1994 for description of these data; and also Venugopal and Foufoula-Georgiou 1996 for an extensive wavelet analysis of these storms).

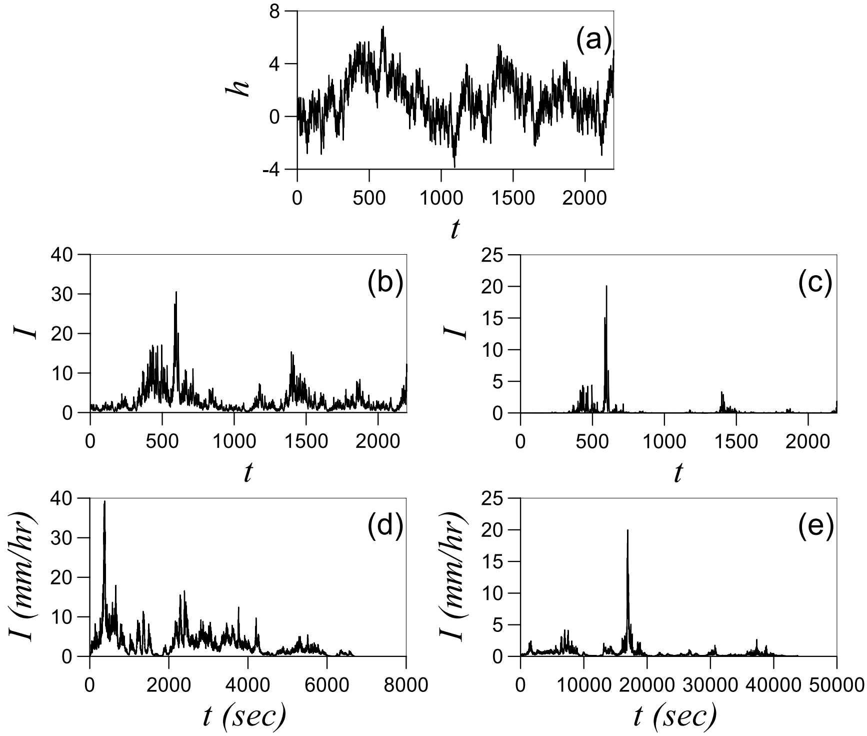

Fig. 6.3. (a) Temporal evolution of the simulated surface $h(x_0, y_0, t)$ at one arbitrary point (x_0, y_0); (b) temporal evolution of the simulated rainfall intensity $I(x_0, y_0, t)$ at point (x_0, y_0) and for $A = 0.5$, $B = 0$; (c) same as (b) but for $A = 1.3$, $B = -6$; (d) and (e) rainfall intensity $I(x_0, y_0, t)$ for two November 1, 1990 storms (storm A and B, respectively) in Iowa City. Notice the similarity between the simulated and the actual rainfalls ((b) resembles (d), and (c) resembles (e)).

The two-dimensional view of the evolving rainfall field, for three different time moments and two different A values, is presented in Fig. 6.4. One can see in Fig. 6.4 that changing the value of A effectively leads to change in the scale of the rainfall field.

As expected, the logarithm of the simulated rainfall field, e.g. the $h(x, y, t)$ surface, showed spatial scaling. Its increments, $\Delta h(x, y, t)$, scaled in space, and gave a Hurst exponent $H \simeq 0.36$ (Figure 6.5). This is close to the spatial Hurst exponent $H = 0.2 - 0.4$ of the normalized fluctuations of natural rainfall $\Delta I(x, y, t)/I(x, y, t) (\simeq \Delta \log(I(x, y, t)) \equiv \Delta h(x, y, t))$ found by Perica and Foufoula-Georgiou (1995) for a large number of midwestern convective storms, providing thus quantitative validation of the proposed model.

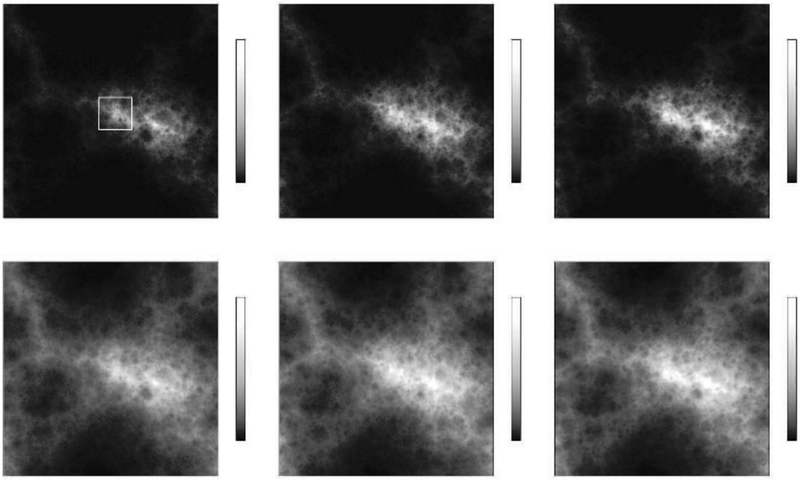

Fig. 6.4. Evolution of the simulated rainfall in two dimensions. The same $h(x, y, t)$ surface (each frame 20 deposition cycles apart from the previous one) is used to produce the rainfall fields in rows (a) and (b) but the A coefficients determining the scale of the rainfall are different: $A = 0.1$ for the upper row, and $A = 0.2$ for the lower row. According to equation (11) the spatial scale in the upper row is $(0.1/0.2)^{1/H} = (0.1/0.2)^{1/0.36} \simeq 7$ times larger than in the lower one. Indeed, one could notice that the rainfall shown in the lower row statistically resembles the boxed smaller part of the upper-row rainfall (box size is 1/7 the size of the whole field). Thus decreasing the A coefficient amounts to zooming into a rainfall field.

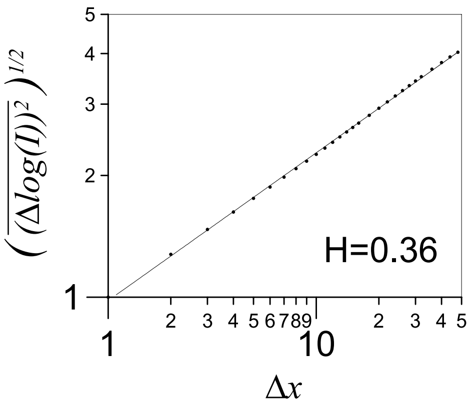

Fig. 6.5. Spatial scaling in the logarithm of the simulated rainfall field, $I(x, y, t)$. The Hurst exponent is $H \simeq 0.36$. It falls within the range $H = 0.2 - 0.4$ found for normalized spatial fluctuations in rainfall (which are approximately the fluctuations in log(rain): $\Delta I/I \simeq \Delta \log(I)$) by Perica and Foufoula-Georgiou (1996).

The logarithm of the simulated rainfall also showed temporal scaling as expected theoretically from the model. The evolving height of any point of the h-surface, $h(x_0, y_0, t)$ behaved as a trail of a fractional Brownian motion, with Hurst exponent H_t = 0.21 (see Figure 6.6a). We compared this result with the behavior of log (rainfall intensity) at a point, for several rainfalls in Iowa. Although not all the logarithmed rainfalls showed good temporal scaling, some of them did. Their Hurst exponent was $H_t = 0.1 - 0.35$, in agreement with the model (which produces $H_t = 0.21$). For example, the temporal scaling in $\log(I(t))$ of the November Iowa storm presented in Fig. 6.3e is displayed in Fig. 6.6b and shows $H_t = 0.29$. The presence of spatial and temporal scaling in the $h(x, y, t)$ surface implies dynamic scaling in the log of simulated rainfall field (the $\log(I(x, y, t))$ surface), with the dynamic scaling exponent $z = H/H_t \simeq 1.7$.

The physical meaning of z is that any feature at the scale L possesses a characteristic lifetime of order $t \sim L^z$, i.e., the feature disappears and/or is replaced by another feature in this time scale. Thus for rainfall, this would mean that a raincell twice as large as another one would dissipate approximately 3.2 (=$2^{1.7}$) times slower, or that it would take a time proportional to $L^{1.7}$ to create or dissipate a rain cell of size L. This result agrees with the dynamic scaling found in Venugopal et al., (1999) for midwestern convective storms (the values of z reported were approximately 1.7). It is noted that in Venugopal et al. (1999) the dynamic scaling exponent was estimated by finding the space-time renormalization of the form $t \sim L^z$ under which the pdfs of $\Delta \log I(L,t) \equiv \log I_L(\tau + t) - \log I_L(\tau)$ remained invariant, where, $I_L(\tau)$ denotes the rainfall intensity at spatial scale L and time instant τ, and t represents the time lag over which the rainfall evolution is measured.

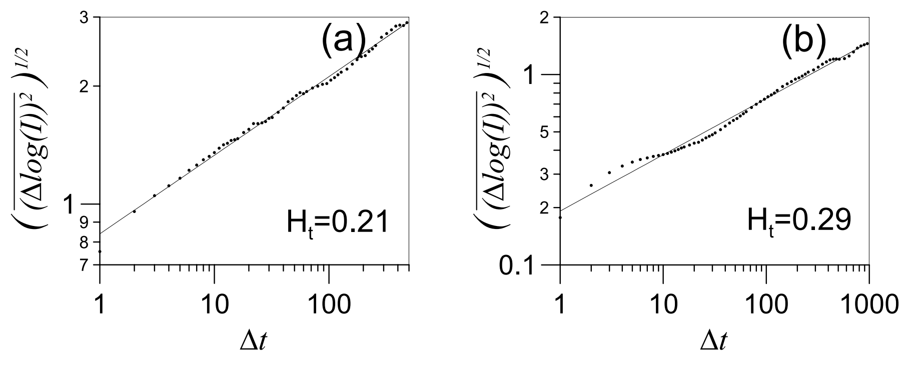

Fig. 6.6. (a) Temporal scaling in the logarithm of the simulated rainfall field $I(x, y, t)$. The Hurst exponent is $H_t \simeq 0.21$. (b) Temporal scaling in the logarithm of the rainfall intensity for the November 1, 1990 B storm in Iowa City. The Hurst exponent is $H_t \simeq 0.29$, close to the exponent produced by the model.

3.2 Nonstationarity and intermittency in rainfall

Whereas the whole rainfall field produced by the proposed model is stationary, its parts are not. Indeed, preserving in time the spatial average of $h(x,y,t)$ over the whole surface does not lead to preserving these averages for an arbitrarily chosen part of the surface. This alone can emulate **nonstationarity** in rainfall. The variations in the average height of a part of the $h(x,y,t)$ surface act as a modulator of the rainfall field intensity $I(x,y,t)$ in that area.

Since at any given instant, different parts of the $h(x,y,t)$ surface have different spatial averages, the rainfall field $I(x,y,t)$ in some of these parts may turn out to be below the sensitivity of the gages (the "dry" areas) and in some other parts above it (the "wet" areas). This creates **spatial intermittency** in the simulated rainfall.

When, as a result of random variation, a point (x_0, y_0) of the evolving $h(x,y,t)$ surface "sinks" deep enough, its exponential mirror, the $I(x,y,t)$ intensity at this point can become very low. We consider time periods over which the rainfall intensity is lower than the sensitivity of a rainfall gage at a given point, as dry periods for this point. When this point "floats up", the next rainfall starts. Thus, the model is capable of reproducing not only a particular rainfall event, but also **temporal intermittency** in rainfall. The temporal intermittency is closely related to the spatial intermittency, both being due to the same features of the generating process.

The model accommodates spatial and temporal intermittency in rainfall by treating dry areas or dry periods as rainfall with very low intensity. Our point is that although some arbitrariness in the sensitivity of the rain gage introduces an element of subjectivity in qualifying a space-time patch as dry or wet, this just reflects the subjectivity existing in real life. Indeed, a sensitive gage could detect rainfall while a less sensitive gage would indicate absence of rain. It should be noted, however, that the exponentiation procedure can easily produce rainfall intensities which are below any reasonably high sensitivity of a gage. Such a period will be dry from the point of view of any observer. Also, it should be mentioned that a space-time patch can be wet or dry depending on its spatial and temporal scale. For example, somebody's hand (which can feel individual drops) put out for a few seconds may not catch any rain drops while a "gage" which is as sensitive as the hand but has a larger size and/or is exposed over a longer time period would signal presence of a low-intensity rainfall.

4 Conclusions

A new class of models for modeling space-time rainfall was introduced as a simple alternative to existing multifractal models. These models are easy to simulate and still have the potential to reproduce important spatial and temporal characteristics of observed rainfall fields over a large range of spatial and temporal scales. These models are based on exponentiation of nonlinear Langevin-type surfaces and represent a rich class within which rainfall of different meteorological environments can be modeled.

In this paper, a simple case was presented in which the standard KPZ model with uncorrelated white noise was used. This model was shown to reproduce well the range of the spatial and temporal scaling exponents empirically documented for midwestern convective storms. Also, by appropriate selection of a model parameter A, the intermittency of space-time rainfall as a function of scale can be reproduced. Since the $h(x,y,z)$ surface is multiplied by the factor A before exponentiation (to produce the rainfall field $I(x,y,t)$) this coefficient affects the spatio-temporal intermittency in the modeled rainfall. In particular, as demonstrated in equation (11), increasing A amounts to increasing spatial and temporal scales. For example, one can notice "dry areas" in Fig. 6.2c obtained using $A = 1.0$, and their absence in Fig. 6.2b obtained from the same $h(x,y,t)$ surface but with $A = 0.5$. Similarly, "dry periods" are present in Fig. 6.3c and absent from Fig. 6.3b. Thus the factor A in the model can be chosen to match the intermittency of a real rainfall field at given spatial and temporal scales.

Although possible advection of the rainfall field imposed by wind has not been presented in this article, the model can easily reproduce it. For that, the rainfall field needs to be shifted in the (x,y) plane with a rate corresponding to the wind velocity.

The model needs further development guided by comparison to real space-time rainfall data, including data on spatial and temporal intermittency. It is noted that, although the parameters A and B can be easily adjusted to reproduce the desired scale and intermittency, the H and H_t exponents describing the space-time structure of the $h(x,y,t)$ surface (and, correspondingly, of the $I(x,y,t)$ rainfall field) cannot be tuned easily. This is because, as most other fractal models, the Langevin-type models exhibit scaling universality, in our case with $H \simeq 0.36$ and $H_t \simeq 0.21$. Although these values pretty much agree with the scaling exponents found in midlatitude convective rain, other scaling exponents (e.g., see Foufoula-Georgiou, 1998 and Foufoula-Georgiou and Venugopal, 2000 for numerically simulated storms and tropical rainfall) can be reproduced using correlated or power-law noise, (e.g., see Medina et al. 1989; Lam and Family 1991; and Zhang 1990), or quenched noise i.e., noise that depends explicitly on h and x but not time t (see chapter 5 of Barabasi and Stanley 1995 and references therein). This issue deserves further investigation.

Acknowledgments

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