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Channel Network Source Representation Using Digital Elevation Models

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Methods for identifying the size, or scale, of hillslopes and the extent of channel networks from digital elevation models (DEMs) are examined critically. We show that a constant critical support area, the method most commonly used at present for channel network extraction from DEMs, is more appropriate for depicting the hillslope/valley transition than for identifying channel heads. Analysis of high-resolution DEMs confirms that a constant contributing area per unit contour length defines the extent of divergent topography, or the hillslope scale, although there is considerable variance about the average value. In even moderately steep topography, however, a DEM resolution finer than the typical 30 m by 30 m grid size is required to accurately resolve the hillslope/valley transition. For many soil-mantled landscapes, a slope-dependent critical support area is both theoretically and empirically more appropriate for defining the extent of channel networks. Implementing this method for overland flow erosion requires knowledge of an appropriate proportionality constant for the drainage area–slope threshold controlling channel initiation. Several methods for estimating this constant from DEM data are examined, but acquisition of even limited field data is recommended. Finally, the hypothesis is proposed that an inflection in the drainage area–slope relation for mountain drainage basins reflects a transition from steep debris flow-dominated channels to lower-gradient alluvial channels.

INTRODUCTION

The problem of identifying the actual extent of the channel network from digital elevation models (DEMs) is of considerable geomorphic and hydrologic importance, given the widespread use of DEMs in simulation models. Physically based models of hydrologic and erosional processes require differentiation between the runoff generation and erosion mechanisms that operate on hillslopes and in channels. For example, channel network extent directly affects the simulated hydrologic response of a catchment because it determines both hillslope travel distances and network link lengths and thus hydrologic response functions [e.g., *Rodriguez-Iturbe and Valdes*, 1979; *Gupta et al.*, 1980]. Consequently, accurate estimation of network source locations is important for accurate runoff prediction using these methods. Identification of network sources is thus of fundamental importance in landscape-scale geomorphic and hydrologic analyses.

A variety of methods exist for automatically extracting channel networks from DEMs [e.g., *Mark*, 1983, 1988; *O'Callaghan and Mark*, 1984; *Band*, 1986; *Morris and Heerdegen*, 1988; *Smith et al.*, 1990], but little attention has been paid to the accuracy of network source representation. The most common method of extracting channel networks from DEMs is to specify a critical support area that defines the minimum drainage area required to initiate a channel [e.g., *Band*, 1986, 1989; *Zevenberger and Thorne*, 1987; *Morris and Heerdegen*, 1988; *Tarboton et al.*, 1988; *Lammers and Band*, 1990; *Gardner et al.*, 1991]. In practice, this threshold value often is selected on the basis of visual similarity between the extracted network and the blue lines depicted on topographic maps. However, *Morisawa* [1957], and later *Coffman et al.* [1972], demonstrated that blue line networks provide only a poor representation of channel networks observed in the field because they do not depict first-order channels, as well as many second- and third-order channels. Other methods used to simulate network source locations include the degree of contour indentation, or crenulation [e.g., *Strahler*, 1952; *Morisawa*, 1957; *Lublowe*, 1964; *Smart and Surkan*, 1967; *Howard*, 1971; *Abrahams*, 1980], and a minimum slope [e.g., *Shreve*, 1974]. Two general methods have been used to simulate network sources in digital terrain models: a constant threshold area [e.g., *O'Callaghan and Mark*, 1984; *Band*, 1986; *Mark*, 1988; *Tarboton et al.*, 1991] and a slope-dependent critical support area [e.g., *Dietrich et al.*, 1992, 1993]. This paper critically examines these approaches and suggests modifications to existing procedures and interpretations.

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THEORETICAL BASIS FOR ESTIMATING CRITICAL SUPPORT AREAS

Models that predict either constant or slope-dependent thresholds assume different criteria for channel initiation. Assuming that channel heads represent a transition in the dominant sediment transport process implies a constant critical support area, whereas assuming that channel heads represent an erosional threshold leads to a slope-dependent critical support area. These models and their development are discussed in greater detail elsewhere [*Dietrich and Dunne*, 1993; *Montgomery and Dietrich*, 1994]. Below we outline key assumptions and briefly discuss model development and application to the problem of predicting network source locations from digital elevation data.

Constant Threshold

The logic behind a constant critical support area descends from ideas originally developed by *Gilbert* [1877, 1909]. Essentially, the hypothesis holds that slope-dependent sediment transport on hillslopes gives rise to convex slopes, whereas discharge- and slope-dependent sediment transport in channels gives rise to concave slope profiles. A series of workers developed this hypothesis into the proposition that channel heads correspond to the transition from convex to concave slope profiles [e.g., *Kirkby*, 1971, 1980, 1986; *Smith and Bretherton*, 1972; *Tarboton et al.*, 1992].

These models are based on coupling the continuity equation with distinct sediment transport laws for hillslope and fluvial processes. In general, sediment transport may be considered to reflect the availability of sediment for transport and the competence of the transporting medium. Thus in most cases, the sediment transport rate (q_s) will be a function of the slope (s), discharge (q), and, if transport capacity exceeds sediment supply, the production rate of transportable material (p):

$$q_s = f(s, q, p) \tag{1}$$

For soil-mantled landscapes, many workers exclude p from (1) and formulate a general sediment transport equation as

$$q_s = kq^n s^m \tag{2}$$

where k, n, and m are constants [e.g., *Kirkby*, 1971, 1980, 1986; *Ahnert*, 1976, 1987, 1988; *Tarboton et al.*, 1992].

In most landscape evolution models, sediment transport by hillslope and fluvial processes involve different parameter values. Models for hillslope sediment transport [e.g., *Davison*, 1889; *Gilbert*, 1909; *Culling*, 1963; *Kirkby*, 1971] generally assume that $n = 0$, and the most widely used formulation for geomorphic modeling is the linear diffusion model ($m = 1$):

$$q_s = k_1 s \tag{3}$$

where k_1 is an erosional diffusivity. Fluvial sediment transport is a function of both discharge and slope, and most workers assume that $n > 1$ and $m \approx 2$ for channel processes.

A number of workers have shown that equilibrium slope profiles for a slope-dependent transport law are convex, which requires that slope increases with either distance from the drainage divide, or drainage area [e.g., *Davison*, 1889; *Gilbert*, 1909; *Culling*, 1963; *Kirkby*, 1971, 1980, 1986]. Transport laws with both slope and discharge dependency, on the other hand, result in concave equilibrium slope profiles [e.g., *Gilbert*, 1877; *Kirkby*, 1971, 1980, 1986; *Smith and Bretherton*, 1972]. *Smith and Bretherton* [1972] further showed that concave slope profiles occur where

$$\partial q_s/\partial q > q_s/q \tag{4}$$

and that convex slope profiles result when the opposite inequality is satisfied. They then showed that concave surfaces are unstable to lateral perturbation, whereas convex surfaces are stable. *Smith and Bretherton* [1972] also proposed a general sediment transport law for both fluvial and hillslope processes where

$$q_s = k_1 s + k_2 q^n s^m \qquad n > 1 \qquad k_1, k_2 > 0 \tag{5}$$

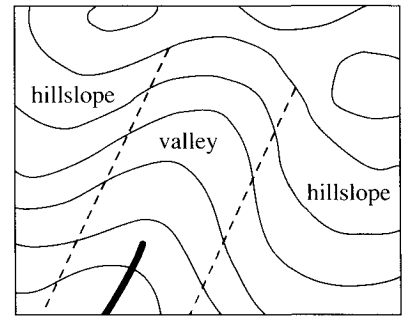

Fig. 1. Schematic topographic map illustrating distinctions between hillslopes, valleys, and channels. Hillslopes are areas of topographic divergence; valleys are areas of topographic convergence. Dashed lines indicate transition from hillslopes to valleys. A channel (thick black line) is a morphologic feature defined by sediment transport concentrated within definable banks. Channels typically begin some distance down valley axes from drainage divides.

and argued that channel heads correspond to the transition from convex to concave slopes. *Kirkby* [1980, 1986] substituted contributing area per unit contour length (a) for discharge in (4) and also suggested that the channel head was coincident with a transition from convex slopes where $\partial q_s/\partial a > q_s/a$ to concave slopes where $\partial q_s/\partial a < q_s/a$. *Tarboton et al.* [1992] further proposed that the channel head corresponds to a transition from hillslopes where $\partial s/\partial a > 0$ to channels where $\partial s/\partial a < 0$. However, the transition from convex to concave slope profiles commonly coincides with the transition from divergent to convergent topography, suggesting that these models are more appropriate for representing controls on valley development than for channel initiation [*Dietrich and Dunne*, 1993; *Montgomery and Dietrich*, 1994] (Figure 1). No data are presently available, however, with which to directly test this hypothesis.

Slope-Dependent Threshold

Models for a slope-dependent critical support area assume that the channel head represents an erosional threshold, an assumption similar to *Horton*'s [1945] hypothesis for drainage network evolution. This assumption requires that the channel head represents a change in sediment transport processes, rather than a spatial transition in process dominance. Another key assumption in this approach is that the channel head is associated with erosion initiation, an expectation that will not be met, for example, in landscapes where sheetwash erosion occurs. Field observations in humid, soil-mantled landscapes, however, generally support the association of channel heads with a change in sediment transport processes [*Dietrich and Dunne*, 1993; *Montgomery and Dietrich*, 1994].

A general model for sediment transport with a threshold control on channelization is given by

$$q_s = k_1 s \qquad a < a_{cr} \tag{6a}$$

$$q_s = k_1 s + k_2 q^n s^m \qquad a \geq a_{cr} \tag{6b}$$

where a_{cr} is the critical contributing area per unit contour length for channel initiation [*Dietrich and Dunne*, 1993; *Montgomery and Dietrich*, 1994]. Essentially, this formulation argues that sediment transport occurs by slope-

dependent processes upslope of a critical contributing area per unit contour length, whereas discharge-dependent processes also transport sediment downslope of the channel head.

The critical contributing area per unit contour length required to initiate a channel may be estimated from simple models of channel initiation processes. Field observations and measurements suggest that these processes include overland flow, seepage, piping, and landsliding [e.g., *Kirkby and Chorley*, 1967; *Dunne*, 1980; *Jones*, 1987; *Montgomery and Dietrich*, 1988, 1989, 1994; *Dietrich and Dunne*, 1993]. Models for channel initiation by overland flow and shallow landsliding predict inverse relations between critical support area and local slope.

Channel initiation by overland flow may be assumed to occur where the basal shear stress of the flow (τ_b) exceeds the critical shear stress of the ground surface (τ_{cr}). For a steady state rainfall intensity (q_r) a laminar flow model predicts that the critical contributing area required for $\tau_b > \tau_{cr}$ is given by

$$a_{cr} = C/(\tan\theta)^2 \qquad C = f(\tau_{cr}^3,\ q_r^{-1}) \tag{7}$$

where θ is the local slope [*Dietrich et al.*, 1992, 1993; *Montgomery and Dietrich*, 1993]. Thus smaller drainage areas are needed to initiate channels on steeper slopes. Rearranging (7), channels may be defined using the criterion of $a(\tan\theta)^2 \geq C$. The absolute value of the proportionality constant C depends on both ground surface (τ_{cr}) and climatic (q_r) properties.

An analogous model for channel initiation by shallow landsliding is derived from combining a model for shallow throughflow and the infinite slope stability model. This simple hybrid model for cohesionless soils predicts that

$$a_{cr} = (T/q_r)\sin\theta\,(\rho_s/\rho_w)[1 - (\tan\phi/\tan\theta)] \tag{8}$$

where T is the soil transmissivity, ρ_s and ρ_w are the bulk density of the soil and water, respectively, and ϕ is the friction angle of the soil [*Montgomery and Dietrich*, 1994]. This relation predicts a nonloglinear relation between the critical support area and local slope and is only valid for steep slopes where $\tan\theta \geq [(\rho_s - \rho_w)/\rho_s]\tan\phi$.

The available field data on channel head locations in landscapes where overland flow and landsliding are the dominant channel initiation processes support the general form of (7) and (8) [*Montgomery and Dietrich*, 1994]. In mountainous landscapes, the transition from landslide to overland flow-dominated channel initiation occurs at $\tan\theta \approx 0.5$. However, for most of this paper we will consider only channel initiation by overland flow, as is expressed by (7). The reader interested in channel initiation in steeper landscapes is referred to discussions presented elsewhere [*Dietrich et al.*, 1986, 1987; *Montgomery and Dietrich*, 1989, 1994; *Dietrich and Dunne*, 1993]. Identification of an appropriate value for C is a major impediment to implementing the overland flow model for channel network extraction from DEMs, as this parameter should vary with both rainfall and critical shear stress of the ground surface, the latter reflecting both soil properties and the type and density of vegetation cover.

APPLICATION OF METHODS

Each of the preceding hypotheses may be correct under different circumstances and over different time scales. In landscapes where rainsplash and sheetflow occur, for example, channel heads may, indeed, coincide with a spatial transition in process dominance. In many soil-mantled landscapes, however, a threshold model for channel initiation is more appropriate. *Montgomery and Dietrich* [1994] proposed that a continuity based approach, like that employed to argue for a constant a_{cr}, is most appropriate for modeling valley development over geologic time (e.g., 10^4–10^6 years), whereas a threshold-based approach is most appropriate for modeling channel head locations over shorter, geomorphic time scales (e.g., 10^2–10^3 years). Thus these hypotheses need not be viewed as competing ideas, rather they are complementary. Below we examine methods for implementing these criteria and test their appropriateness for locating channel network sources against high-resolution DEM data from an area in which the channel network has been mapped in the field.

The effect of spatial variability of slope on channel network extent is one primary difference between channel networks defined by constant and slope-dependent thresholds. Using a slope-dependent threshold, drainage density is greater in steeper portions of a catchment, as is found in natural landscapes. For the case of a catchment with little spatial variability in slope, the constant and slope-dependent threshold methods converge and predict similar channel networks for the same mean source-area size. In relatively small, or homogeneous basins the channel networks defined by these methods may not differ significantly. Differences between the two methods should increase with basin size and geologic complexity.

Constant Threshold

The theory equating a channel head with the transition from slope-dependent hillslope transport to slope- and discharge-dependent fluvial transport [e.g., *Smith and Bretherton*, 1972; *Kirkby*, 1980; *Willgoose*, 1989; *Tarboton et al.*, 1992] predicts that the channel head is associated with a change in the relation between local slope and drainage area (or discharge). *Tarboton et al.* [1991, 1992] developed this criterion into a method for determining network source locations from digital elevation data. We show below that, as proposed, their method is inconsistent with the theory and predicts anomalously large hillslope sizes. We also show that a similar approach, reformulated in more appropriate terms, provides a reasonable estimate of the scale of the transition from convergent to divergent topography, or the hillslope/valley transition.

The method proposed by *Tarboton et al.* [1991, 1992] consists of the following operations. First, a critical support area is assumed and a channel network is extracted from a DEM. This is then repeated for a number of support areas. Next, one generates a plot of link slope (the average channel slope between two confluences) versus the drainage area at the downstream end of the link for each link in the extracted channel networks. Finally, the individual link values are averaged and the "appropriate" critical support area is determined from an inflection in the composite drainage area–slope relation for the averaged data.

Each of these steps presents either conceptual or procedural problems. First, the method identifies an inflection in the drainage area–slope relation, rather than the reversal from a positive to inverse relation predicted by the theory.

Although a reversal is seen in some data sets, the drainage area at which it occurs depends upon the support area initially assumed to extract the channel network (see Figure 9 of *Tarboton et al.* [1991]). Consequently, *Tarboton et al.* [1991] chose instead to relate an inflection observed in some data sets to the scale of channel initiation and offered an argument to explain that this inflection represented the relation expected from "noisy" DEM data.

The use of the link slope presents further problems. First, it requires that a channel network be extracted from the DEM. In other words, the search for an appropriate support area is constrained to be within a previously defined channel network. Furthermore, link slopes introduce systematic biases into the data. This occurs because the link slope is defined as the elevation difference between the upper and lower confluences bounding the link divided by the length of the link. For a slope profile along which the gradient is not constant, the average slope of a channel link is a function of the length of the link. However, the length of individual links within a catchment depends upon the threshold area used to define the channel network. Link slopes then depend on the threshold used to extract the channel network, which probably explains the threshold size control on the reversal in the drainage area–link slope relation.

Another problem with the proposed method is a consequence of common flow-routing algorithms in grid-based digital terrain models. The traditional method of directing flow from one grid cell, or pixel, into one of its eight neighbors does not allow for the representation of divergent flow [e.g., *Moore et al.*, 1988; *Freeman*, 1991]. In essence, flow is allowed to converge into valleys, but not to diverge on hillslopes, resulting in preferential flow partitioning along the cardinal and diagonal directions of the grid matrix in divergent topography [e.g., *Freeman*, 1991; *Fairfield and Leymarie*, 1991]. This artefact makes it difficult to represent, let alone distinguish, the transition from convergent to divergent topography. A number of newer algorithms for representing divergent flow have been developed [*Freeman*, 1991; *Quinn et al.*, 1991; *Cabral and Burges*, 1992], but are not yet widely implemented.

This artefact of representing hillslopes with linear, rather than divergent, flow allows a simple check on the hillslope lengths implied by the method of *Tarboton et al.* [1991, 1992]. The drainage area contributing to a pixel at the base of a divergent hillslope will be equal to the width of the pixel multiplied by the upslope distance to the drainage divide. Conversely, the hillslope length will equal the drainage area at the hillslope/valley transition divided by the pixel width. The inflection in the link slope plots of *Tarboton et al.* [1991, 1992] occurs at drainage areas of 10^5 to 10^6 m^2. For a pixel size of 30 m, this implies hillslope lengths on the order of 3.3–33 km. Field surveys indicate that typical hillslope lengths are on the order of tens to hundreds of meters, but in low-gradient semiarid landscapes hillslope lengths may approach 1 km (T. Dunne, unpublished data, 1993). Thus the drainage area–slope relation noted by *Tarboton et al.* [1991, 1992] is related to something other than the hillslope/valley transition.

To be more consistent with the theory outlined above, we formulate our analyses in terms of local slopes, which in a grid-based DEM may be best approximated by the slopes for individual grid elements, or pixels. Figure 2 shows plots of drainage area versus local slope for two U.S. Geological

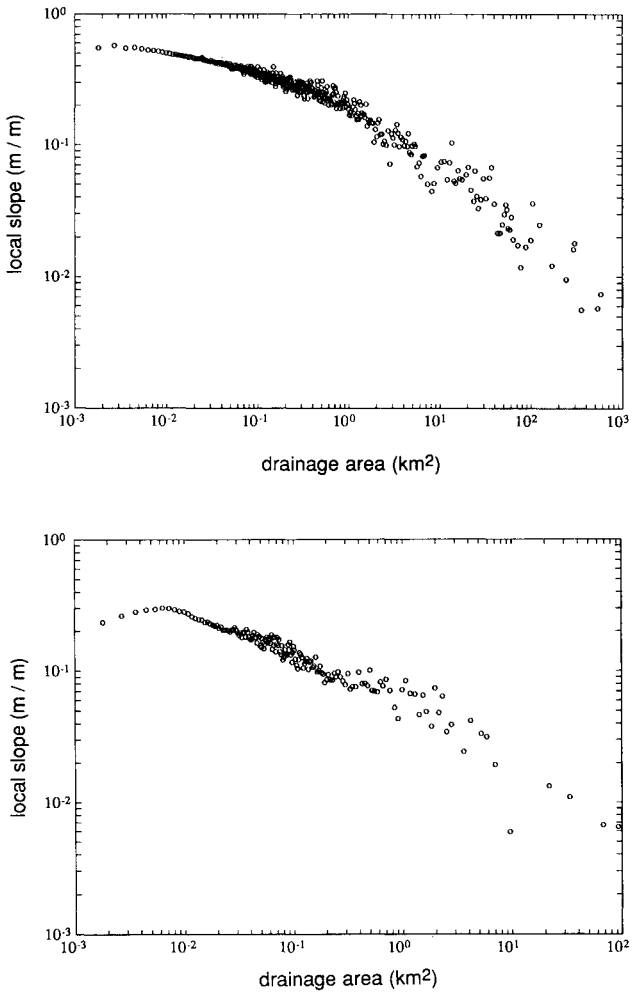

Fig. 2. Plots of local slope versus drainage area for averaged data from all pixels for USGS 30 m DEMs of (top) South Fork Smith River, California and (bottom) Schoharie Creek, New York.

Survey (USGS) 30 m DEMs. Prior to averaging, there is tremendous scatter apparent in the data, but the averaged plot for the South Fork Smith River, California (Figure 2(top)) reveals an inflection at a drainage area of approximately 1 km^2. Plots constructed with the link-averaging technique also show this inflection. The Smith River is in rugged mountainous terrain and a typical 1 km^2 watershed in this area is shown in Figure 3. The drainage area associated with the inflection in the drainage area–slope relation (the "hillslope" scale of *Tarboton et al.* [1991, 1992]) roughly corresponds to the outlet of this basin, which contains a magnitude 17 valley network. The reversal predicted by the theory is not readily apparent in Figure 2(top), but could be reflected in the two smallest data points at a drainage area of about 2×10^{-3} km^2, which would imply a hillslope length of 67 m. Extensive field experience in the coastal mountains of southern Oregon and northern California provides the basis for concluding that this provides a reasonable estimate of hillslope lengths in this area.

A similar plot of drainage area versus local slope for the Schoharie Creek catchment reveals a reversal at an area of 6×10^{-3} km^2, but no inflection at larger areas (Figure 2(bottom)). The pixel width of 30 m thus implies a hillslope length of 200 m. The Schoharie Creek watershed is a

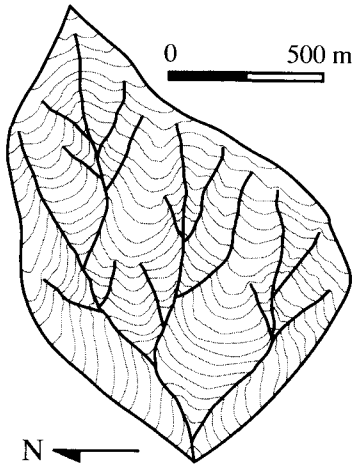

Fig. 3. Portion of USGS Ship Mountain 7.5′ topographic quadrangle showing a typical 1 km^2 watershed in the South Fork Smith River drainage basin. Solid lines indicate valley network defined by areas of convergent topography.

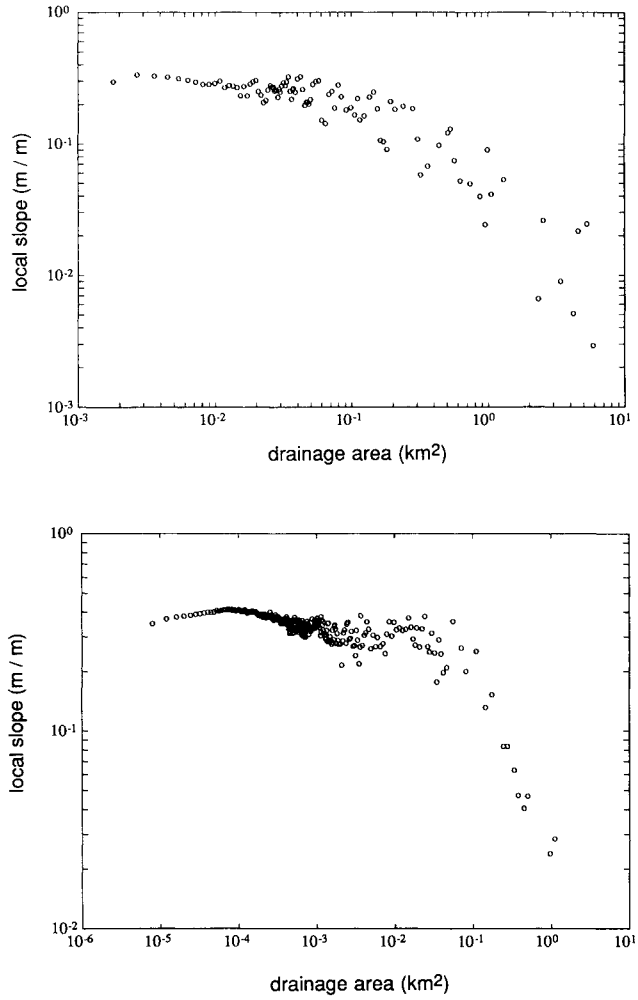

Fig. 4. Plots of drainage area versus local slope for averaged data from the Tennessee Valley area of Marin County, California for (top) a portion of USGS Point Bonita 7.5′ quadrangle DEM data and (bottom) high-resolution data for a subbasin of the area covered by the USGS data.

low-gradient area and, although we have no field experience in this region, this estimate of the hillslope length seems reasonable.

It appears then that we must explain two distinct changes in the drainage area–slope data derived from digital elevation models; a reversal at very small drainage areas, and an inflection in the relation at local slopes of about 0.2 to 0.3. We chose to examine this question with high-resolution DEMs (2 m grid) of areas in which the channel network previously was mapped in the field [*Montgomery and Dietrich*, 1989, 1992]. The DEM of the Tennessee Valley area near San Francisco, California was generated from low-altitude stereo aerial photographs for use in geomorphic process modeling [*Dietrich et al.*, 1992, 1993]. Consequently, the DEM was generated at a resolution sufficient to capture the divergent form of the hillslopes in this catchment. To allow comparison with more typical resolution data, we also examine results obtained using the USGS 30 m grid size DEM for a larger area that contains the catchment covered by high-resolution data. For both data sets, drainage areas and local slopes were calculated using the eight neighbor method for flow partitioning.

The plot of the averaged drainage areas versus local slope for the 30-m data (Figure 4(top)) exhibits an inflection at a drainage area of about 10^{-1} km^2 and a reversal suggested by the averaged values for individual pixels at a drainage area of about 3×10^{-3} km^2. The hillslope length implied from the inflection is 6.6 km; that implied by the reversal is 100 m. For comparison, the length of the entire watershed covered by the high-resolution data is about 1 km. A similar plot from the high-resolution DEM also exhibits a pronounced inflection at a drainage area of about 10^{-1} km^2 (Figure 4(bottom)). In this case, however, a well-defined reversal in the trend of the averaged data occurs at 10^{-4} km^2, corresponding to a hillslope length of 50 m.

We used a digital terrain model capable of representing topographic divergence to test whether the transition from divergent to convergent topography coincides with the reversal apparent in the drainage area–slope relation. The digital terrain model TOPOG [*O'Loughlin*, 1981, 1986] was used to divide the catchment into a series of topographic elements defined by the intersection of contours and orthogonal flow lines and to calculate the slope and contributing area per unit contour length for each element. In a modification of the criteria used by *Dietrich et al.* [1992], we divided elements into divergent and convergent classes based on the ratio of the length of the lower and upper contours bounding the element. The lower contour is longer for divergent elements and shorter for convergent elements. Plots of drainage area versus slope for elements in both the Tennessee Valley area and similar high-resolution data from Mettman Ridge in coastal Oregon [*Montgomery and Dietrich*, 1994] both reveal a general reversal from a positive to a negative relation coincident with the transition from divergent to convergent slopes (Figure 5). In both cases, there is tremendous data scatter and most of the data cluster around the hillslope/valley transition. The transition from divergent to convergent elements for both data sets occurs at a contributing area per unit contour length of 30–50 m, essentially equal to the hillslope length suggested by analysis of the high-resolution grid-based DEM. Field mapping of channel networks and surveyed slope profiles in both of these

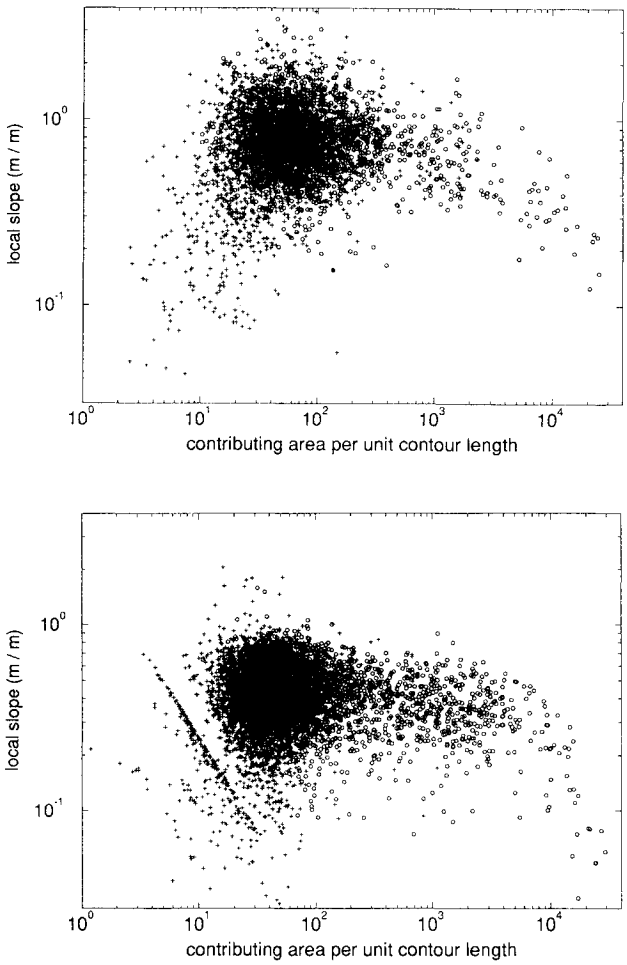

Fig. 5. Plots of local slope versus contributing area per unit contour length for each topographic element generated by TOPOG [*O'Loughlin*, 1986] from high-resolution DEM data from (top) Mettman Ridge, Oregon and (bottom) Tennessee Valley, California. Data from divergent elements are shown as crosses, and data from convergent elements are shown as circles. Linear band of data in Figure 5(bottom) is an artefact of flow tube architecture and has no physical significance.

areas [*Montgomery and Dietrich*, 1988, 1989, 1992] confirms that these values provide reasonable estimates of hillslope lengths, indicating that when used with sufficient resolution, a constant support area is appropriate for defining the hillslope/valley transition.

Slope-Dependent Threshold

The available field data from arid to temperate environments indicate that the drainage area required to initiate a channel is a function of the local slope, except in locations where bedrock properties control channel head locations [*Montgomery and Dietrich*, 1994]. Channel heads in Tennessee Valley define a slope-dependent transition from unchanneled to channeled valleys (Figure 6), implying that the channel network could be defined given the appropriate threshold. Although landsliding dominates channel initiation on the steepest slopes in this area, the regression of the critical drainage area (A_{cr}) against local slope for data from channel heads yields

$$A_{cr} = 1790 \tan \theta^{-1.84} \qquad r^2 = 0.68 \tag{9}$$

where A_{cr} is in square meters [*Montgomery*, 1991]. This relation similar to that predicted by (7).

The field-mapped channel network greatly exceeds the network defined by blue lines on the USGS 7.5′ quadrangle (Figure 7), as the blue line network depicts only two first-order channels in this magnitude 36 watershed. In contrast, the channel network defined by $A_{cr} \geq 2000 \tan \theta^{-2}$ reasonably approximates the field-mapped channel network, indicating that (7) may be used to delineate the channel network, given appropriate constraints on C.

Based on field data from three study areas in California and Oregon, *Montgomery* [1991] reported that

$$C \approx 10^6/R_a \tag{10}$$

where C is in square meters and R_a is mean annual rainfall in millimeters. Although this relation suggests a convenient method for estimating an appropriate value of C, it is important to recognize that the critical shear stress is also a major determinant of C. For example, field data from an arid, dissected alluvial fan in Nevada (J. Repka, unpublished data, 1993) indicate significantly smaller source areas than in any of the semiarid to humid study areas mapped by *Montgomery and Dietrich* [1988, 1992], documenting that changes in the critical shear stress due to variations in ground cover may dominate the value of C. Thus simple correlation of C with mean annual rainfall, as in (10), is unjustified. Unfortunately, there is no theory for predicting the value of C directly from either drainage area or slope, the primary topographic attributes that may be derived from a DEM.

Dietrich et al. [1993] presented a method for approximating C that involves estimating τ_{cr} and calibrating q_r against observed hydrologic response. For the Tennessee Valley area, the steady state rainfall required to saturate convergent areas of the landscape provides a reasonable estimate of q_r [*Dietrich et al.*, 1993]. Given that C is a function of the third power of τ_{cr}, the predicted extent of the channel network is extremely sensitive to the assumed value of τ_{cr} [*Dietrich et al.*, 1993]. When τ_{cr} and q_r may be estimated with some confidence, then this technique may be used to simulate channel network source locations. In many potential applications, however, appropriate values of these parameters will be unknown.

Several other approaches allow estimation of channel network extent using a slope-dependent threshold. *Mont-*

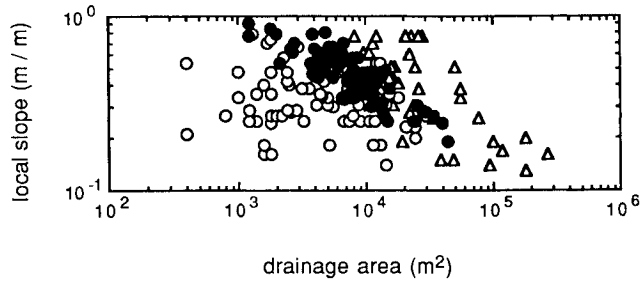

Fig. 6. Plot of drainage area versus local slope for data from field mapping in the Tennessee Valley catchment. Triangles are from channels, solid circles are from channel heads, and open circles are from unchanneled valleys. Data from *Montgomery and Dietrich* [1992].

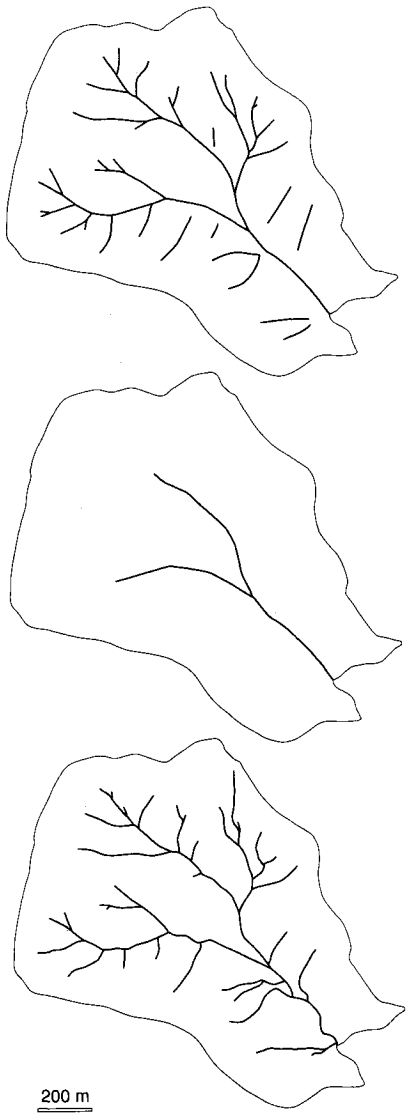

Fig. 7. Comparison of channel networks determined for the Tennessee Valley watershed by field mapping (top), blue lines depicted on USGS Point Bonita 7.5′ quadrangle (middle), and A_{cr} (tan θ)^2 = 2000 m^2 (bottom).

gomery and Dietrich [1992] presented evidence that, in a landscape where channel head locations may reflect a long-term average position, the variance in channel head locations effectively defines the limit to convergent topography, and thus the hillslope size. They showed that portions of the landscape where a (tan θ)^2 was just less than necessary to sustain a channel essentially surround the valley network. Thus if channel networks are defined using smaller and smaller thresholds, then eventually numerous channels should branch from the sides of channels and generate a "feathering" of channels extending into planar or divergent hillslopes (Figure 8). Consequently, one criterion for defining the extent of the channel network is to use the smallest value of C that does not result in development of significant feathering along low-order headwater channels.

We calculated interior-link, exterior-link, and source-basin lengths and areas for channel networks extracted from the Schoharie Creek and Tennessee Valley data using different thresholds to examine whether statistical properties of channel networks are useful for estimating C. *Abrahams* [1984] reviewed previous network studies and concluded that the ratio of the mean interior to mean exterior link length (L_i/L_e) was approximately equal to unity in natural channel networks. Analyses of ratios of interior and exterior link lengths from networks defined with differing threshold areas indicate that these properties do not change systematically with the imposed threshold (Figure 9(top)). Although this ratio varies, it remains generally close to unity. This reflects the interdependence of the number and lengths of network links. A smaller source area results in an almost equal increase in the number of interior and exterior links and decreases the mean length of both populations. Consequently, this ratio is rather insensitive to the source area used to define the network and therefore does not provide useful constraints on network extent.

Link area ratios (A_e/A_s and A_i/A_s), on the other hand, may be more useful. *Montgomery* [1991] found that for channel networks in small basins mapped in the field mean exterior-link drainage area was just less than twice the mean source area ($A_e/A_s \approx 2$). The mean exterior and interior link areas for the Schoharie Creek catchment converge to the empirical values of twice the source area size at C = 8,000 to 16,000 m^2 (Figure 9(bottom)). For the Tennessee Valley catchment, this occurs at C = 500 m^2 (Figure 9(bottom)). Probability distribution functions (pdfs) of interior-link, exterior-link, and source-basin lengths and areas vary with the threshold used to define the channel network. For the C values examined, the pdfs for source and network properties in the Schoharie Creek catchment appear most similar for C = 16,000 m^2 and the analogous pdfs for networks extracted from the high-resolution Tennessee Valley DEM appear most similar at C = 1000–2000 m^2. Although there is no unique value of C predicted by examination of network statistical properties, such procedures may provide rough estimates of appropriate values to use for channel network extraction from DEMs.

The approaches discussed above entail the implicit assumption that the channel network is in long-term equilibrium with the land surface upon which it is developed. In reality, channels at any given time could start anywhere within the valley network. In badland landscapes, for example, channels may extend onto hillsides in a manner resembling the feathering discussed earlier. In some landscapes extensive dry valleys record significant climatic variations. Furthermore, previous land-sculpting processes, such as glaciation, may control channel network architecture. Realistically, if the objective is an accurate description of the contemporary channel network extent, then there is no proven substitute for collecting some field data on channel head locations. A threshold of the form a_{cr} (tan θ)^2 could be fit to even minimal field data and then used to extrapolate the channel network extent in similar areas.

DISCUSSION

While a reversal in the drainage area–slope relation correlates with the hillslope/valley transition, what controls the inflection observed in the averaged drainage area–slope relation for some data sets? Where it is present, the inflection in the drainage area–slope plots of Figures 2 and 4

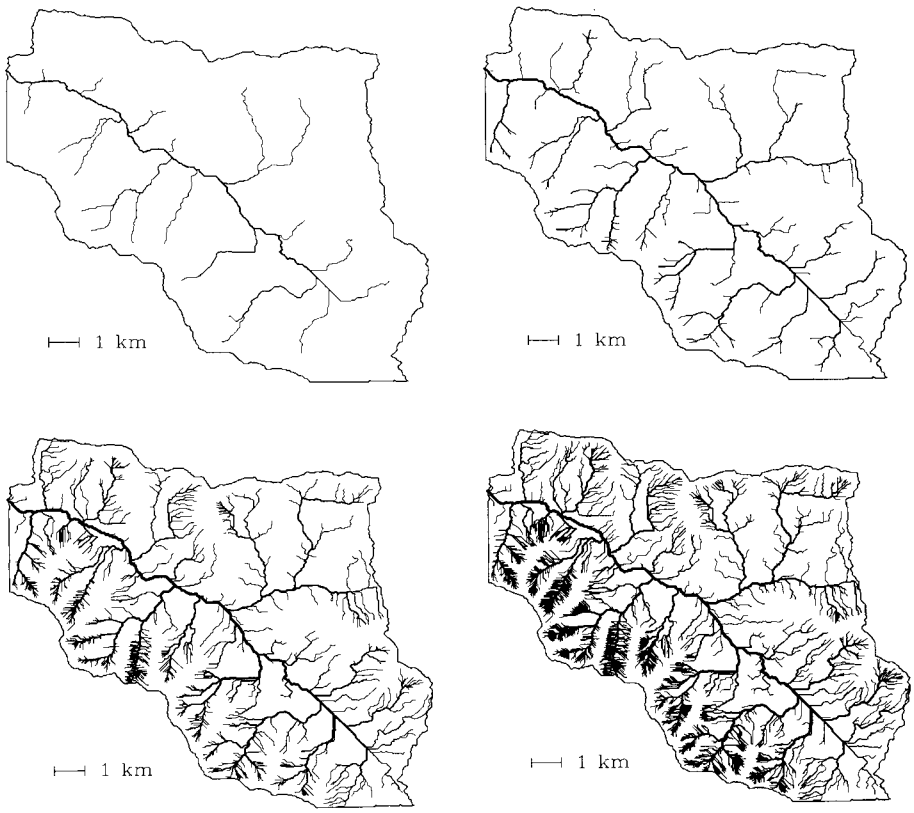

Fig. 8. Maps of the Schoharie Creek channel network defined using A_{cr} (tan θ)^2 = (top left) 64,000 m^2, (top right) 16,000 m^2, (bottom left) 4,000 m^2, and (bottom right) 2,000 m^2. Note the feathering along low-order channel in the lower (steeper) portions of the catchment in Figures 8(bottom left) and 8(bottom right).

occurs at gradients between 20 and 30%. Each of the inflections identified in the figures presented by *Tarboton et al.* [1991] occur in this same gradient range. In the Tennessee Valley study basin, the inflection also is apparent in the analysis of high-resolution DEMs (Figure 4), indicating that it is not a consequence of poor DEM resolution. The drainage area at which this inflection occurs ($\approx 10^{-1}$ km^2) correlates well with the transition from either colluvium-floored channels, or those with a discontinuous veneer of alluvium overlying bedrock, to lower-gradient channels within alluviated valleys. Field mapping of channels in mountain drainage basins in Oregon and Washington indicates that a transition from debris flow-dominated to alluvial channels occurs in this gradient range (D. R. Montgomery, unpublished data, 1993) and corresponds to a change in the drainage area–slope relation observed within the channel network. *Seidl and Dietrich* [1992] report a change in the drainage area–slope properties of tributary junctions derived from topographic maps of the Oregon Coast Range at gradients of about 20% and argue that this transition reflects a change in the dominant erosional mechanism from debris flow to fluvial processes. We suggest that the inflection observed in the drainage area–slope relation derived from DEMs reflects this transition in valley incision processes.

The results presented above suggest a schematic illustration of landscape partitioning into drainage area and slope regimes that define hillslopes, unchanneled valleys, and debris flow-dominated and alluvial channels (Figure 10). Hillslopes are defined by topographic divergence and hillslope size may be approximated by a constant drainage area. Unchanneled valleys occupy the lowest gradients for a given drainage area. The boundary between unchanneled valleys and fluvial channels is defined by a slope-dependent threshold that reflects both critical shear stress and climate. The data available at present suggest that channels on slopes in excess of 20–30% are debris flow-dominated. Thus we would not expect to see this inflection in the averaged drainage area–slope relation for low to moderate gradient landscapes. This model for landscape partitioning suggests that boundaries between debris flow-dominated and alluvial channels and between hillslopes and valleys can be estimated directly from DEMs, but that the extent of the channel network cannot. Other considerations, such as those developed above, are necessary to approximate the locations of channel heads from DEMs.

CONCLUSIONS

Theories for valley development and channel initiation respectively predict constant and slope-dependent critical support areas. Both theories are supported by field data. The extent of topographically divergent hillslopes, and thus the extent of the valley network, corresponds to a change in sign of the relation between local slope and contributing area per unit contour length. This transition is readily derivable from digital elevation data, but a higher-resolution grid size may be required than the 30-m data commonly available. The extent of debris flow-dominated channels also can be deter-

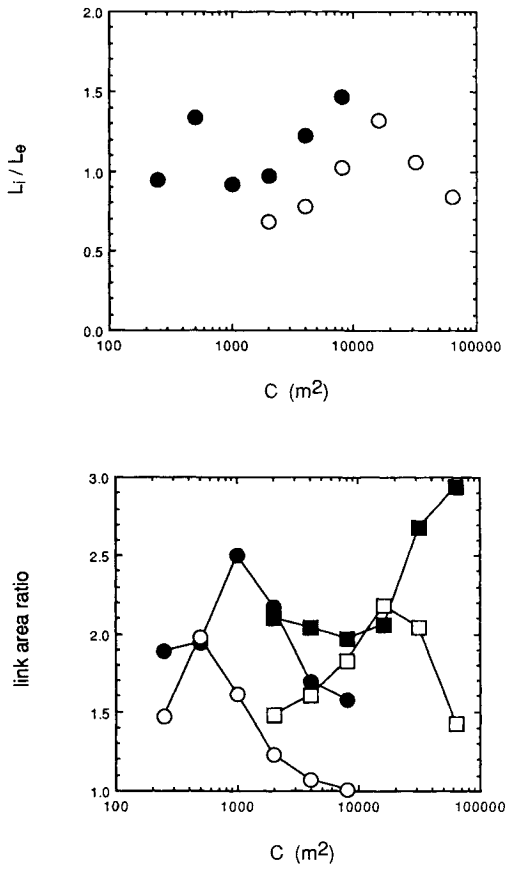

Fig. 9. (Top) Ratio of mean interior to exterior link length versus C $[A_{cr}(\tan\theta)^2]$ for Schoharie Creek (open circles) and Tennessee Valley (solid circles). (Bottom) Ratio of mean link area to mean source area for exterior (solid) and interior (open) links from Schoharie Creek (squares) and Tennessee Valley (circles).

mined from an inflection in the drainage area–slope relations derived from a DEM. In contrast, the extent of the contemporary channel network cannot be directly determined from drainage area–slope relations extracted from DEMs. Field data, even if limited, on the drainage area–slope relation for channel heads is the best method for determining appropriate values of parameters defining channel network extent.

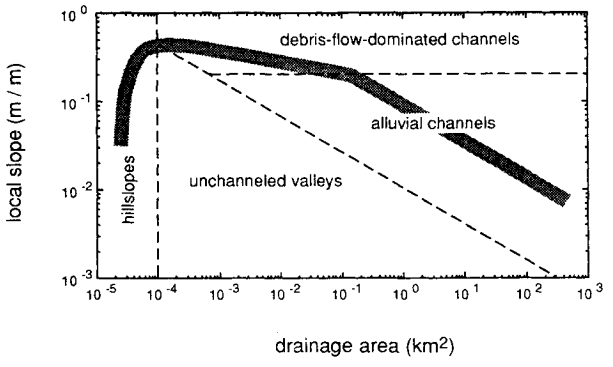

Fig. 10. Schematic illustration of relations between drainage area and local slope depicting hillslope/valley transition and channel initiation criteria. Note that the trend of the averaged data (wide shaded curve) for a catchment will vary between landscapes and watersheds.

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