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Introduction to special section on Stochastic Transport and Emergent Scaling on Earth's Surface: Rethinking geomorphic transport—Stochastic theories, broad scales of motion and nonlocality

Efi Foufoula-Georgiou[1] and Colin Stark[2]

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[1] In many geomorphic transport systems, the time and length scales of motion vary widely: particles can be trapped for both short and long periods of time and they can travel large or small distances in very short intervals of time. To model such systems we need fresh conceptual and mathematical formalisms. The goal of this collection of papers is to challenge existing thinking in geomorphic transport by putting forward new ideas and theories for environmental fluxes, from particle transport in a single stream, to landslide debris mobilization, sediment and water transport on hillslopes, dynamic transport on river networks, and interpretation of sedimentary deposits over geologic time. Advanced stochastic theories of transport are proposed, the notion of nonlocal flux is introduced, and fractional advection-diffusion equations are explored as possible models of geomorphic transport.

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1. Introduction

[2] In November 2007 a working group meeting entitled Stochastic Transport and Emergent Scaling on the Earth's Surface (STRESS) was convened in Lake Tahoe, Nevada. Its scope was to bring together geomorphologists, hydrologists, mathematicians, and physicists with the goal of rethinking the mathematical treatment of transport processes on the Earth's surface. The specific questions asked were (1) How can we reconcile observed patterns and organization (from sand dunes, to landslides, sedimentary deposits, hillslope profiles, and sediment transport in rivers) with theories and dynamical models that can reproduce these patterns? (2) Are geomorphic transport laws based on the notion of a locally derived flux limited in some fundamental sense, and is the notion of a nonlocal flux (flux determined by conditions at some distance from the point of interest) a physically viable alternative? (3) How can we relate microscale and macroscale dynamics in stochastic transport theories and in predictive models?

[3] The papers in this special issue provide some insight into these questions. They stretch the envelope of geomorphic modeling by introducing new mathematical theories and models of transport, providing new explanations of old data, and posing alternative hypotheses to explain process from form. They also open new questions for future research.

2. Challenging Old Theories: Notion of Nonlocality

[4] Current geomorphic transport laws for landscape evolution are formulated as partial differential equations framed around approximations of the physics of advection and diffusion/dispersion: in particular, assumptions are made that facilitate the integration of processes from microscales in time and space to geomorphic model scales. Such assumptions are inconsistent with transport processes in which significant contributions to the total flux come from events across a broad span of magnitude and frequency.

[5] For example, the advection-dispersion equation (ADE) is based on the classical definition of divergence of a vector field. The classical notion of divergence maintains that as an arbitrary control volume shrinks, the ratio of total surface flux to volume must converge to a single value. However, if a considerable fraction of the total sediment flux during the period of observation arises from a flux of particles from far upstream, then the classical divergence theorem fails. Instead, a divergence associated with a finite volume, and defined as the ratio of total flux to volume, is more appropriate. However, by increasing the arbitrary volume a greater heterogeneity in the medium properties and in the physical processes contributing to transport is sampled, and the degree of resulting variability is bound to depend on scale. Thus, the ratio of total flux to volume does not remain

[1]Saint Anthony Falls Laboratory and National Center for Earth-surface Dynamics, Department of Civil Engineering, University of Minnesota, Minneapolis, Minnesota, USA.

[2]Lamont-Doherty Earth Observatory, Earth Institute at Columbia University, Palisades, New York, USA.

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constant but varies with the size of the volume. As a result, the classical diffusion equation is no longer self-contained with a closed form solution at all scales. Within the limits of classical theory, the best that can be done is to assume that the total flux to volume ratio is constant within small ranges of scales, allowing one to talk about an "effective" scale-dependent dispersion coefficient. Another approach is to depart altogether from the classical theory.

[6] Recently, the physics of advection and dispersion has evolved beyond describing classical phenomena to include materials that exhibit variability from large to very small scales, power law velocity distributions, chaotic dynamics, and slow reactive transport (e.g., see the review paper of *Schumer et al.* [2009]). Fractional calculus treatments of advection and diffusion/dispersion capture nonclassical behavior in a simple and elegant form (for an insightful physical interpretation of fractional derivatives see *Podlubny* [2002]). For example, changing the second derivative in the diffusion equation to a fractional derivative of order less than two yields a model for superdiffusion, in which particles spread faster than the classical diffusion equation predicts. Changing the first-order time derivative in that same equation to a fractional derivative of order less than one models subdiffusion. Superdiffusive models are connected with power law particle jump lengths; subdiffusive models emerge from power law waiting times between jumps.

[7] Several techniques have been proposed in the subsurface transport literature to tackle the problem of scale-dependent dispersivity which arises for similar reasons, namely, the presence of inhomogeneities at all scales and the wide range of length and time scales of motion (see discussion and references in the work of *Schumer et al.* [2009]). The treatment of surface transport faces similar challenges, i.e., time- and scale-dependent flux statistics, presence of heterogeneities at all scales, scale invariance and power law spectra of landscapes (in analogy with fractal porous media in the subsurface), and yet geomorphic flux laws that can accommodate such behavior are not available. The contributions in this issue are a step in this direction.

3. New Theories and New Perspectives

[8] The contributions in this volume can be broadly classified as addressing the following three geomorphic transport problems: (1) bed load transport in rivers [*Ganti et al.*, 2010; *Bradley et al.*, 2010; *Ancey*, 2010; *McElroy and Mohrig*, 2009; *Hill et al.*, 2010], (2) transport on hillslopes [*Foufoula-Georgiou et al.*, 2010; *Tucker and Bradley*, 2010; *Furbish et al.*, 2009a, 2009b, *Harman et al.*, 2010; *Stark and Guzzetti,* 2009], and (3) transport in erosional-depositional systems and river networks [*Voller and Paola*, 2010; *Zaliapin et al.*, 2010; *Schumer and Jerolmack,* 2009]. A summary of these developments is presented below.

3.1. Bed Load Transport in Rivers

[9] Despite considerable research over the past several decades, the problem of accurate estimation of bed load sediment transport in rivers remains unsolved. One of the main challenges lies in the fact that the motions of individual particles happen at random, rendering the process of transport a stochastic process. Along these lines, contributions in this issue relate to the development of new stochastic discrete or continuous formulations of transport that can explicitly account for stochastic behaviors such as large variations in particle displacement and long times spent in immobile phase.

[10] *Ganti et al.* [2010] reconsider the problem of tracer dispersal in rivers and argue that long leading tails in tracer concentration are to be expected in certain cases where the step length distribution of particle movements is heavy tailed [see also *Stark et al.*, 2009]. Starting with an active layer formulation of the probabilistic Exner equation they show that the continuum equation describing the tracer concentration in this case takes the form of a fractional advection-dispersion equation. By identifying the probabilistic Exner equation as a forward Kolmogorov equation, they also propose a stochastic model describing the evolution of tracer concentration and show that the classical (normal) and fractional (anomalous) advection-diffusion equations arise as long time asymptotic solutions of this stochastic model. More data are needed to fully verify such a model based on particle-scale and macroscale statistics.

[11] *Bradley et al.* [2010] revisit a 50 year old tracer experiment in which the tracer plume exhibits behavior not possible to be explained with classical transport models, namely, anomalously high fraction of tracers in the downstream tail of the distribution, a decrease of detected tracer mass over time and enhanced particle retention near the source. They propose a fractional advection-dispersion equation and a two-phase transport model (which partitions mass into detectable mobile and undetectable immobile phases) and show an impressive agreement with observations.

[12] *McElroy and Mohrig* [2009] note that the movement of bed material associated with bed deformation is not accounted for in standard methods of calculating sediment flux and propose a framework for calculating that portion of the flux in sandy bed rivers (which they term deformation flux). They note differences between laboratory and real river systems in the statistics of the bed deformation rates and define normalized metrics for comparing systems of different size. They also note the time dependence (power law scaling) of the deformation flux in sand bed rivers, not explainable by classical theories of advection-dispersion, motivating the exploration of fractional dispersion models that can explain such scaling behavior.

[13] *Ancey* [2010] examines the influence of randomness in bed sediment flux on the initial genesis of bed forms, and shows how strong fluctuations in flux can arise even in the absence of heavy-tailed probability distributions of streambed sediment exchange. A Markovian, birth-death process model of sediment entrainment is developed and cast into a stochastic form of the Exner equation. In the large number limit, he shows that the model admits a Fokker-Planck representation, simplifying subsequent analysis. Derivations of the stochastically varying number of particles in motion and of the coupled bed height are provided, allowing prediction of the scaling of the variance of model bed topography with time.

[14] *Hill et al.* [2010] consider the problem of modeling bed load transport in gravel bed rivers which exhibit a broad range of particle sizes. Based on a series of carefully controlled flume experiments, they document an exponential distribution of the travel time of entrained particles of a given size,

with the parameter of the distribution (mean travel distance) depending on both particle size and shear stress. In real settings, the convolution of the distributions of travel distances and particle sizes is shown to yield a power law distribution, which requires reconsideration of standard diffusion models and introduction of superdiffusive models of transport.

3.2. Transport on Hillslopes

[15] Sediment transport on hillslopes forms an area of active research both theoretical and experimental. Typical models available to date include standard diffusive models which consider a linear or nonlinear formulation of flux based on local slope or other local attributes such as soil depth. The contributions in this issue address some important elements of hillslope transport related to stochasticity in the diffusion coefficient to incorporate rain splash effects or dependence on soil thickness, extension to a nonlocal flux formulation (in a discrete or continuous framework) to incorporate large scales of particle motion, reformulation of the kinematic wave equation for hillslope subsurface transport, and a stochastic theory for landslide-driven erosion.

[16] *Stark and Guzzetti* [2009] present a physically based stochastic theory for landslide-driven erosion. The proposed model describes a simplified process of rupture propagation, slope failure and debris mobilization, and it reproduces the probability distributions observed for landslide source areas and volumes, including their power law tail scaling. The peak (rollover) and tail scaling of the distributions are explained in terms of the relative importance of cohesion over friction in setting slope stability, allowing thus for a physical interpretation. Numerical experiments validate the analytical results and document the sensitivity of the model to parameterization. The interplay of river incision and hillslope steepening in adjusting the landslide magnitude-frequency is interpreted in physical and statistical terms.

[17] *Furbish et al.* [2009a] revisit the problem of soil grain transport by rain splash and formulate it as a stochastic advection-dispersion process. One of their innovations rests on the explicit separation of the grain activity probability (determined by the rain storm intensity and soil properties at weather time scales) from the physics of the grain motions. They perform rain splash experiments to confirm that gradients in raindrop intensity are as important as gradients in grain concentration and surface slope in affecting overall transport. Their result points to the importance of the ecological behavior of desert shrubs as “resource islands” (temporary storage zones of soil derived from areas surrounding the shrubs) and the implication that this behavior can have for land-surface evolution modeling. The proposed formulation provides a general framework for transport and dispersal of any soil material moveable by rain splash, including nutrients, seeds and soil-borne pathogens.

[18] *Furbish et al.* [2009b] probe the physical justification of the linear slope-dependent transport formulation. Balancing the particle fluxes that tend to loft a soil with the gravitationally driven particle settling, they show how a slope-dependent transport relation emerges with, however, a statistical description of the diffusion-like coefficient. This coefficient involves the active soil thickness as a fundamental length scale that provides the minimum length scale over which measurement of the surface slope is meaningful. This in turn implies that the diffusion-like linear slope-dependent model (soil flux proportional to the depth-slope product) is applicable at scales larger than the disturbance scales producing the transport. The formulation is consistent with observations of topographic profiles of hillslopes evolving by soil creep and by transport associated with biomechanical mixing. However, the theory does not explain the nonlinear flux-slope relations observed in many systems.

[19] *Tucker and Bradley* [2010] are concerned with transport on hillslopes exhibiting a broad distribution of grain motion length scales. They examine, in a simple discrete particle-based model, relations between grain motion dynamics, bulk transport rates, and hillslope morphology, and they illustrate conditions under which standard local gradient theory is not appropriate. They show that a nonlinear relationship between flux and local gradient emerges from their discrete model formulation at steep slopes and make a preliminary exploration of continuum generalizations based on a probabilistic form of the Exner equation. They provide insightful discussion on the notion of nonlocal flux computation and how high-probability, long-distance particle motions violate the assumption embedded in many commonly used local gradient-based geomorphic transport laws, calling for extensions.

[20] *Foufoula-Georgiou et al.* [2010] propose a nonlocal formulation of sediment flux on hillslopes to account for the wide range of particle displacement lengths related to disturbance processes. This formulation computes flux at a point not only as a function of local topographic attributes, such as slope, but also as a function of topography upslope of the point of interest. They show that such a formulation leads to a continuum constitutive law that takes the form of fractional diffusion. The model predicts a hillslope equilibrium profile that is parabolic in shape very close to the ridge top and becomes power law downslope, with an exponent equal to the nonlocality model parameter. Furthermore, they show that a nonlinear relationship between sediment flux and local gradient emerges from this linear nonlocal model and that the model reproduces, with a single parameter, the natural variability of sediment flux found in real landscapes.

[21] *Harman et al.* [2010] revisit the problem of subsurface transport in hillslopes with heterogeneous conductivity fields. They argue that, in such cases, variations in the downslope velocity of impulses induce a nonpiston type flow response (piston response would arise from impulses starting at different locations but moving at a constant speed). Assuming heavy tails in the velocity distribution of those impulses, they invoke the notion of subordination (replacing real time with a random time representing the time that impulses spend in motion). As a result they recast the standard kinematic wave equation into a subordinated kinematic wave equation appropriate for modeling flow response in heterogeneous hillslopes. They evaluate their model under different degrees of heterogeneity and link the statistical parameters of the heterogeneous random fields and the parameters of the subordinator, implying that the subordinator can eventually be parameterized by physical measurements of hillslope properties.

3.3. Transport in Erosional-Depositional Systems and River Networks

[22] *Zaliapin et al.* [2010] develop simple theories of dynamic transport on river networks. They introduce the

concept of a "dynamic tree" to describe transport of fluxes on a topological static tree representing the river network. They show that the corresponding dynamic trees exhibit self-similarity, albeit with different parameters than the underlying static trees, providing thus the possibility of developing process-specific dynamic scaling frameworks. They also report a "phase transition" in the dynamics of river networks indicating a time (or equivalently length) scale at which the connectivity of the system reaches a critical point; that is, the system acts as a single cluster. Analysis of three real river networks indicates a possible universality and points to the need for further analysis to understand how this framework can be used for stochastic flux propagation and for scaling of dynamic processes operating on river networks.

[23] *Schumer and Jerolmack* [2009] provide a novel interpretation of the field-documented observation that sediment deposition rate decreases as a power law function of the measurement interval. They argue that this phenomenon is the result of the heavy tailed distribution of non-deposition periods and use limit theory and Continuous Time Random Walk (CTRW) models to estimate the actual average deposition rate from observations of the surface location over time. Their analysis highlights that caution has to be exercised in attributing observed changes in accumulation rates through time to real changes in the rates of erosion and deposition. The consequences of these finding for interpreting the stratigraphic record in terms of climate variability are important.

[24] *Voller and Paola* [2010] put forth the observation that laboratory experiments of aggrading rivers, driven by subsidence or base level rise, display profiles that deviate from those expected from standard diffusion models. They propose a fractional diffusion model which accounts for non-Fickian sediment transport in systems where the length scale of significant sediment extraction is comparable to the scale range of the channel pattern behavior. They point out that these length scales seem well separated in natural systems but not in laboratory systems. This distinction may explain discrepancies between laboratory and natural system profiles and has implications for modeling.

4. Closing Remarks and Open Problems

[25] The 15 papers in this volume present new ideas for modeling transport on the Earth's surface from tracer and bed load transport in rivers, to hillslope transport, to the complexities of mixed erosional/depositional systems, and to transport along the whole river network. They explore stochastic formulations that account for the deformation of bed forms as they contribute to sediment flux, the erosional impact of spatially and temporally variable raindrops as they contribute to the ecology and geomorphology of hillslopes, theories for explaining the power law distributions of landslide areas and volumes, and theories that take into account the broad range of scales participating in transport. Several papers revisit old data sets and show that predictions from generalized transport laws agree with observations more closely than predictions based on classical theories. A few papers attempt to make connections between microscale (particle scale) dynamics and macroscale statistics and note that parameters of the macroscale models can be resolved from physical observables as opposed to empirical fitting. Emphasis is placed on parsimonious parameterizations, that is, on models that can explain the observed structure and variability with few parameters. The idea of nonlocality in flux computation is discussed in several papers and fractional advection-dispersion formulations or discrete space-time models are proposed.

[26] Several open problems have emerged from the research presented in these papers. First, the physical motivation of nonlocal transport laws and the data needed to more directly estimate model parameters and discriminate between local and nonlocal hypotheses are areas of future study. Also, stochastic formulations that invoke particle-scale statistics explicitly or implicitly require new kinds of data, such as statistics of particle movement, to be tested and validated. The same applies to models that consider bed form deformation as a diffusion problem. The idea of extending well known transport models via time subordination is compelling and awaits more exploration: such an approach will have application in the modeling of environmental fluxes in which "time in motion" rather than "clock time" is relevant and where time can therefore be treated as a random variable. The exploration of Continuum Time Random Walk (CTRW) models as discrete counterparts of continuum formulations has to be further studied, and extensions of those models to two dimensions awaits development.

[27] A problem with all local geomorphic transport laws is that they yield scale-dependent sediment flux since the local slope and curvature are scale (resolution) dependent. As such, closures are needed to incorporate the effect of subgrid scale variability and render the model coefficients scale independent [see, e.g., *Passalacqua et al.*, 2006]. An open problem for future research is to examine whether nonlocal transport models naturally overcome the problem of scale dependence, as this becomes an issue of increasing concern with the availability of high-resolution topographic data.

[28] Theories for the transport of fluxes on river networks where the heterogeneity of the input (e.g., spatially variable precipitation which dynamically changes over time, or discrete fluxes that are injected at only a portion of the nodes of the network) await further development such that scaling relations incorporating both the topology of the network and the dynamics of the driving process are considered. Finally, models for transport in erosional/depositional systems that capture the large range of scales of motion, and the use of these models for the interpretation of the stratigraphic record (e.g., apparent scale-dependent erosion rate), require new data to be rigorously tested and validated. The outcomes could have important implications for deciphering climate variability from stratigraphy.

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E. Foufoula-Georgiou, Saint Anthony Falls Laboratory and National Center for Earth-surface Dynamics, Department of Civil Engineering, University of Minnesota, 2 Third Ave. SE, Minneapolis, MN 55414, USA.

C. Stark, Lamont-Doherty Earth Observatory, Earth Institute at Columbia University, Palisades, NY 10964, USA.