Chapter 3

Hydrologic Advances in Space-Time Precipitation Modeling and Forecasting

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Abstract. The spatial and temporal rainfall characteristics influence the runoff hydrograph of a catchment, while accurate forecasting of precipitation is crucial to the prediction of floods and flash floods. This study presents a review of recent hydrologic advances in space–time rainfall modeling and forecasting. The strengths and weaknesses of the available models are discussed and directions for further research and improvements are given.

1. Introduction

1.1. Significance of Precipitation

Precipitation is the source component of the hydrologic cycle. As such, it regulates water availability and thus land use, agricultural and urban expansion, maintenance of environmental quality, and, in general, operation and management of water resources systems. Extreme rainfall events, and the floods they produce, determine the design and operation of large hydraulic structures, shape up the land by the drastic geomorphic changes they induce, and are often responsible for loss of property or sometimes lives due to flooding.

1.2. Spatial and Temporal Rainfall Characteristics

The importance of the spatial and temporal rainfall distribution on the runoff hydrograph of a basin has been demonstrated by many studies. For example, Wilson et al. (1979) concluded that the spatial distribution of rain and the accuracy of the precipitation input considerably influence the volume of storm runoff, time–to–peak, and the peak runoff of small catchments.

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Hamlin (1983) and Nicks (1982) reached similar conclusions for drainage basins of less than 3,000 km^2. Tabios et al. (1987) examined the effects of the dynamic characteristics of a storm, e.g., storm orientation and speed, on the streamflow hydrograph and concluded that these characteristics are important and should not be neglected in current design practices. The effect of storm size on the rainfall–runoff relationship was examined by Milly and Eagleson (1982) for relatively large catchments and Milly and Eagleson (1988) for cases where the storm size is of comparable size to the modeled area. Using simple approximations of the spatial rainfall distribution and duration, they concluded that the infiltration–excess runoff of a large area is extremely sensitive to the storm size and areal rainfall distribution and that, in general, spatial variability of precipitation produces increased surface runoff as compared to a uniform rainfall of the same volume. Eagleson (1984) and Eagleson and Wang (1985) stressed the importance of storm to catchment scales on the runoff production. They derived (for an idealized case of circular catchment and circular storm) the first two moments of the catchment area covered by a stationary storm; they found that the expected value of the covered area for most storm and catchment scales of hydrologic interest is much less than one, illustrating the danger of invoking the one–dimensionality assumption in hydrologic studies.

1.3. Precipitation Research

The study of rainfall has been an active area of research in the last decade. Under the lead of the American Geophysical Union, a Precipitation Committee was formed in 1982 composed of hydrologists, meteorologists, physicists, mathematicians, and statisticians. One of the main goals of the committee (AGU, 1984) was to bring together the isolated efforts of all these disciplines and bridge the gap between them. Precipitation studies were reported in several conferences and special sessions of the American Geophysical Union meetings. Collection of articles from two of these meetings were published in Water Resources Research (Vol. 21, No. 8, August, 1985) and Journal of Geophysical Research (Vol. 92, No. D8, August, 1987). The main theme of precipitation research in the last decade has been the understanding of the physical mechanisms producing rainfall and the incorporation of the precipitation dynamics and meteorological and physical evidence of rainfall organization patterns in the structure of stochastic rainfall models. Research in this area has rapidly evolved. One needs only to compare the review article of Court (1979) with that of Georgakakos and Kavvas (1987) to realize the tremendous volume of new developments and new ideas in the last decade and to sense the beginning of an emerging interdisciplinary effort. In the area of real–time hydrometeorology, the first physically–based models were developed that were compatible in scale and form with operational hydrologic models. Thus, a great potential exists in this area for the development of hydrologically useful precipitation models that integrate all available remote and on–site observations at the local hydrologic scale.

The areas of precipitation research can be broadly classified as: (a) precipitation measurement and estimation, (b) precipitation modeling in space and time, and (c) quantitative precipitation forecasting. Our review will concentrate on the areas of statistical precipitation modeling and quantitative precipitation forecasting at spatial and temporal scales of interest to hydrology, i.e., at the basin–scale. The reader is referred to the recent article by Georgakakos and Kavvas (1987) for a review of new developments on precipitation measurement via remote sensors, i.e., radar and satellite. This article is also complementary to our review in the other two areas of modeling and forecasting. The focus of our presentation is towards the operationally useful precipitation models for hydrologic simulation and forecasting and the critical assessment of the state–of–the–art models in terms of limitations and need for further developments.

2. Discussion

2.1. Precipitation Modeling

Rainfall is the result of complex atmospheric processes evolving continuously over space and time. Space–time rainfall modeling, based on the mathematical deterministic description of the underlying atmospheric processes, is extremely complicated and has little operational usefulness in hydrologic applications. The lack of complete understanding of the physical and dynamic processes involved in the formation of precipitation imposes limits on the ability of these models to make deterministic predictions beyond a limited temporal and spatial resolution (Cho, 1985; Cho and Chan, 1987). The need for mathematically tractable descriptions of rainfall for operational purposes has motivated the treatment of rainfall as a stochastic process. In this section, we first review the stochastic rainfall models in time over a single site or over a number of sites. We then review the stochastic rainfall models which describe the rainfall field as a continuous random field in space and time.

2.1.1. Temporal Rainfall Models at a Single Site or at a Number of Sites

Due to the intermittency of the small time–increment rainfall (e.g., hourly or daily), two characteristic processes are of importance: the rainfall occurrence process and the process of the non–zero rainfall amounts. These two processes can be modeled either simultaneously (a compound process) or separately, and then be superimposed. Regarding the modeling approach of the rainfall occurrences, the temporal rainfall models proposed to date can be classified in three main categories: (a) the "wet–dry spell" approach; (b) the discrete time series approach; and (c) the point process models approach (in continuous and discrete time).

The "Wet–Dry Spell" Approach. In the "wet–dry spell" approach, any uninterrupted sequence of wet days or hours defines an "event." Such an occurrence process is completely specified by the probability laws of the length of the wet periods (storm duration) and the length of the dry periods (time between storms). This model structure, with exponential distributions for the lengths of the dry and wet periods, was used by Thom (1958) and Green (1964), among others. Grace and Eagleson (1966) used a Weibull distribution for the wet-period lengths and applied the model to short time increment (on the order of minutes and hours) rainfall occurrences. Other studies using this modeling approach include those of Todorovic and Yevjevich (1969) and Eagleson (1978).

In probabilistic terminology, the wet–dry spell model is an alternating renewal model. The term renewal stems from the implied independence between the dry and wet period lengths, and the term alternating is used to indicate that a wet (dry) period is always followed by a dry (wet) period, i.e., no transition to the same state is possible. In many early studies, such a model with exponential distributions for the dry period lengths was referred to as a Poisson model. This is inaccurate terminology resulting from the assumption that an event, which in this case corresponds to a wet period, occurs instantaneously at the middle or end of the wet period.

A recent study of an alternating renewal model for daily rainfall was reported by Galloy et al. (1981). They used discrete negative binomial distributions for the wet and dry period lengths and implemented the theory of point processes to derive the statistical properties of intervals (times between events) and counts (number of events in a time interval). Also, Roldan and Woolhiser (1982) compared a first–order Markov chain model with an alternating renewal model that had truncated geometric distribution for the wet periods and a truncated negative binomial distribution for the dry periods. They found that the Markov chain model was superior to the alternating renewal model for the four U.S. daily rainfall records studied.

Small and Morgan (1986) derived the relationships between a continuous-time renewal model and a Markov chain model and applied this analysis to daily rainfall occurrences.

There are certain disadvantages associated with the wet-dry spell approach. The first is the definition of independent events or storms (see, for example, Restrepo-Posada and Eagleson, 1982). This problem is even more pronounced in hourly data where wet sequences separated by one or several dry hours may still correspond to the same rainfall event. The second problem stems from the varying duration of the events which requires that the cumulative rainfall amounts corresponding to each event should be conditioned on the duration of the event. The identification and fitting of conditional probability distributions to the rainfall amounts may pose a problem especially with short records and for events of extreme duration. Finally, once the total storm rainfall has been modeled, it has to be redistributed within the wet period (internal storm characteristics); this requires additional statistical information to be extracted from the limited data available. Thus, the wet-dry spell approach seems more appropriate for the study of the external, rather than the fine-scale internal, storm characteristics.

The Discrete Time-Series Approach. The daily (or hourly) rainfall occurrence series can be viewed as a binary series of zeroes and ones, with zero corresponding to a dry day, and one to a wet day. The simplest probabilistic model for such a binary series is the Bernoulli process, which, due to its complete independent structure, is not adequate to describe the clustering present in the short-time-increment rainfall occurrences (see, for example, Smith and Schreiber, 1973). Markov chain models are the next simplest models with a dependence structure and have been extensively used for modeling daily rainfall. Gabriel and Neumann (1957, 1962) used a first-order homogeneous (i.e., constant parameters) Markov chain for the winter daily rainfall occurrences at Tel-Aviv, while Caskey (1963) and Weiss (1964) used a nonhomogeneous (i.e., time varying parameters) Markov chain for several stations in the Northern U.S. Hopkins and Robillard (1964) used a first-order Markov chain for the daily rainfall occurrences in Canada and found that it was not adequate to describe the months with few rainy days. Feyerherm and Bark (1967) showed the inadequacy of a first-order Markov chain in describing the higher-order dependence structure present in daily rainfall, and they proposed a second-order Markov chain for the daily rainfall occurrences in Indiana, Iowa, and Kansas. Wiser (1965) and Green (1965) also concluded that the geometric memory of the first-order Markov chain is not adequate to describe long droughts or long wet spells. Smith and Schreiber (1973) found a nonhomogeneous first-order Markov chain superior to an independent Bernoulli model for the seasonal thunderstorm rainfall in the Southwestern U.S. Woolhiser and Pegram (1979) studied Markov chain models with seasonally varying parameters using Fourier series.

In deciding the order of a Markov chain, Tong (1975) used the Akaike Information Criterion (AIC), while Hoel (1954) presented a likelihood ratio goodness of fit test. Chin (1977) identified the Markov chain orders of 25-year daily rainfall records in the U.S. and illustrated their dependence on the season and geographical location. A recent study by Stern and Coe (1984) contains a nice exposition of Markov chain models of rainfall sequences.

Although Markov chain models provide a simple mathematical representation of the daily rainfall occurrence process and may be adequate for some specific sites and seasons, their Markovian structure cannot describe the long-term persistence (i.e., long wet or dry spells) and the effect of clustering (i.e., higher likelihood of having an event due to an event at a previous time) present in the short time-increment rainfall occurrences. For example, long-term persistence in the daily rainfall may be caused by cyclonic activity persisting during certain seasons (Petterssen, 1969), and clustering may be the result of frontal thunderstorms with a relatively long life cycle (Kavvas and Delleur, 1981).

A more general class of binary discrete time-series models is the class of discrete autoregressive moving average (DARMA) models. A DARMA(p,q) model, where p is the order of the autoregressive and q the order of the moving average component, is a sequence $\{X_n\}$ formed by a probabilistic combination of elements of a sequence $\{Y_n\}$ which is independent and identically distributed (i.i.d.). For the binary DARMA models, Y_n is assumed to be i.i.d. with a Bernoulli distribution, i.e., $P(Y_n = 0) = \pi_0, P(Y_n = 1) = \pi_1$, and $\pi_0 + \pi_1 = 1$. For further details on these models and derivation of their statistical properties, see Jacobs and Lewis (1978a,b). DARMA models for daily rainfall were first used by Buishand (1978) to analyze wet-dry spells in the Netherlands. Chang et al. (1984) further developed and applied DARMA models to daily rainfall sequences in Indiana and reported satisfactory results.

Although DARMA models may be an improvement over Markov chains, in the sense that they can accommodate longer term persistence in a more parsimonious way than a higher-order Markov chain, in our opinion, their disadvantage is the lack of physical motivation for the model structure and also the discontinuous memory they exhibit (see, for example, Keenan, 1980). Also, their mathematical framework seems only to permit derivation of interval properties (i.e., probability distributions of run lengths) and not of counting properties (i.e., distributions of number of events in a time interval). Given that analysis of the second order properties of intervals and counts are in general unequivalent (see, for example, Cox and Lewis, 1978), it is advantageous to be able to use both for model identification and fitting; this can be done effectively in the point process mathematical framework.

Rainfall Amounts Models. Several marginal probability distributions have been proposed for the non-zero daily or hourly rainfall amounts. Todorovic and Woolhiser (1971) and Richardson (1981) used an exponential distribution, but Skees and Shenton (1974) and Mielke and Johnson (1974) suggested that the exponential distribution has a thinner tail than the one observed in daily rainfall amounts. The mixed exponential distribution was explored by Smith and Schreiber (1973), Woolhiser and Pegram (1979) and Roldan and Woolhiser (1982), among others. This last study compared several distributions (chain-dependent and independent exponential, gamma, and mixed exponential) and found, using the Akaike Information Criterion, that these distributions ranked from best to worst as mixed exponential, independent gamma, chain-dependent gamma, and exponential for five daily records in the continental U.S. The gamma distribution has been extensively used (see, for example, Ison et al., 1971; Buishand, 1978; Carey and Haan, 1978) with satisfactory results. Mielke (1973) and Mielke and Johnson (1974) proposed the use of the kappa or generalized beta distribution. Chain-dependent distributions, assuming that the rainfall amounts are independent but that the distribution function depends on whether the previous day was wet or dry, were studied by Katz (1977) and Buishand (1978). The chain-dependent distributions have the disadvantage of overparametrization. If dependence is present in the non-zero rainfall amounts, ARMA models with skewed marginal pdf's can be considered. Such models have been extensively used for streamflow modeling (e.g., Obeysekera and Salas, 1983) and for serially correlated rainfall durations (e.g., Raudkivi and Lawgun, 1972) but not very much for non-zero daily rainfall amounts because of their almost independent structure. Finally, several studies have considered the simultaneous modeling of daily rainfall occurrences and amounts via multiple-state Markov chain models (e.g., Khanal and Hamrick, 1974; Haan et al., 1976; and Carey and Haan, 1978).

The Point-Process Approach. A point process (PP) is a stochastic process which describes the occurrence (position) of "events" in the modeling space (e.g., the time axis, for the temporal rainfall occurrences). When a magnitude or intensity is attached to each occurrence, then the process is called "a marked point process." Waymire and Gupta (1981 a,b,c) present a nice review of the theory of point processes and their relevance and potential for the stochastic modeling of hydrologic systems. A continuous-time PP permits the events to occur anywhere on the time axis, whereas a discrete-time PP permits them to occur only on the marks specified by equally spaced increments, e.g., one day apart. Since rainfall is in reality a continuous-time process recorded over discrete-time intervals, both continuous and discrete

PP's can be explored and appropriately applied. The majority of the statistical literature (e.g., Cox and Lewis, 1978; Cox and Isham, 1980) has dealt with continuous-time PP's. This, together with the appealing idea of describing the rainfall process at the continuous time scale (and thus, by aggregation, at all other sampling scales) has motivated the application of continuous-time PP's in hydrologic modeling. Foufoula-Georgiou and Lettenmaier (1986) illustrate the problems associated with some early applications of the theory of continuous-time PP models to discrete-time rainfall data.

A continuous-time point process framework assumes the existence of an underlying continuous-time rainfall process with outcome that is only observed as the integral of the continuous process over a fixed sampling interval (e.g., hours or days). The problem in such a framework becomes that of inferring the properties of the underlying, unobserved process from the observed discrete data. The important question that naturally arises in such an approach is that of scale dependency. That is, are the parameters of the postulated unobserved continuous-time process consistent when estimated from data at different time scales, such as hourly and daily? Some of the models are clearly scale dependent, while some others seem to yield constant parameters over a range of time scales. Clearly, the models that are scale independent are preferable for two reasons. First, they may provide useful insights into the structure of the underlying rainfall process, especially when a physical meaning can be associated with the scale-independent parameters of the model. Second, these models provide general descriptions of practical value in that a model fitted to hourly rainfall data will guarantee the preservation of the daily rainfall characteristics. Below, we first present a review of the state-of-the-art continuous-time PP rainfall models, discuss their time-scale dependency, and draw comparisons between them. We then proceed with a discussion of the discrete-time PP rainfall models.

The simplest continuous-time point process is the Poisson process in which the events occur completely at random. In a Poisson process, the times between events are independent and exponentially distributed, and the number of events in a time interval is independent and Poisson distributed. The Poisson process has been extensively used to model the rainfall occurrences (see, for example, Todorovic and Yevjevich, 1969; and Gupta and Duckstein, 1975; for some of the early studies). More recently, Rodriguez-Iturbe et al. (1984) have studied two marked Poisson processes. The marks associated with each event are either instantaneous rainfall amounts (Poisson white noise, PWN, model) or rectangular pulses (Poisson rectangular pulses, PRP, model). The pulses are characterized by an intensity and a duration which are assumed to be i.i.d's and independent of each other. For these processes, Rodriguez-Iturbe et al. (1984) derived the moments of the aggregated rainfall amounts at any time scale, T, and used these parameters to fit the model to daily rainfall occurrences in Denver, Colorado and Agua Fria, Venezuela. They observed that both the PWN and PRP models are scale dependent in that the models yielded significantly different parameter estimates when fitted to hourly and daily data. Moreover, the Markovian dependence structure of the PRP model could not reproduce the empirical correlation functions of the hourly data. The inadequacy of the PWN model for short-time increment rainfall was also demonstrated (using the second order properties of a general marked point process) by Foufoula-Georgiou and Guttorp (1986, 1987).

Kavvas and Delleur (1981) observed that the daily rainfall occurrences in Indiana exhibit a clustering which might be satisfactorily modeled by the class of Poisson cluster models and, in particular, by the Neyman-Scott (N-S) models. A N-S process is a two-level process. At the primary level, the rainfall generating mechanisms (RGM) occur according to a Poisson process. Each RGM gives rise to a group, or cluster, of rainfall events. Within each cluster, the occurrence of events is completely specified by the distribution of the number of events and the distribution of their positions relative to the cluster center. Kavvas and Delleur (1981) assumed a geometric distribution for the number of rainfall events in a cluster and an exponential distribution for the distances of events from their cluster centers. Rodriguez-Iturbe et al. (1984) further studied the N–S white noise (NSWN) process, and derived the second-order properties of the accumulated rainfall amounts over different time scales. They found this model superior to the Poisson models for the analyzed hourly and daily rainfall data at Denver and Agua Fria. A more detailed analysis by Valdes et al. (1985) re-examined the time scale sensitivity of the NSWN, PWN, and PRP models by using temporal rainfall series generated from the space–time rainfall model of Waymire et al. (1984). They found that the N–S model performed the best at different time scales but that none of these models was able to preserve the statistics of extremes. Foufoula-Georgiou and Guttorp (1986) observed that since one of the parameters of the N–S model (the one related to the dispersion of events in a cluster) is uniquely determined by the rate of decay of the autocorrelation function of the cumulative rainfall amounts, the NSWN model cannot be time-scale invariant. In addition, these authors examined alternative fitting procedures by employing the properties of the discrete–time occurrence series derived by Guttorp (1986) and pointed out the insufficiency of the second order statistics in identifying the underlying continuous process, at least from daily data. An extensive simulation study illustrating other estimation problems and the properties of estimators for these models was reported by Obeysekera et al. (1987).

Motivated by the inadequacies of the NSWN model, Rodriguez-Iturbe et al. (1987a) introduced the N–S rectangular pulses (NSRP) model and the Bartlett-Lewis (B–L) rectangular pulses (BLRP) model. The rectangular pulse is characterized by a random intensity and duration. The B–L process differs from the N–S process only in the way in which the cells are positioned within a cluster. In the N–S process, the position of the cells is determined from the storm origin according to an exponential distribution. In the B–L process, the intervals between successive cells (and not the distances from the storm centers) are i.i.d. and exponentially distributed. Both the cases of a Poisson and a geometric distribution of the number of cells within a cluster were considered. Based on a detailed analysis of data from Denver, at different aggregation levels, Rodriguez-Iturbe et al. (1987b) concluded that these models are capable of preserving several rainfall characteristics at different levels of aggregation while the parameters of the model remain essentially constant. Also, these models seemed to preserve some extreme order statistics although the probability of the long dry periods was overestimated by both models (see, Rodriguez-Iturbe et al., 1987a). A modified version of the BLRP model was developed by Rodriguez-Iturbe et al. (1987c). This modification consisted of allowing for different characteristics among storms by randomizing several parameters of the distributions of the number of cells per storm, cell positions, and cell durations. These modifications seemed to improve the representation of the extreme events.

A point process model of a different structure was introduced by Smith and Karr (1983). This process, namely the doubly stochastic Poisson process (also known as the Cox process), has a rate of occurrence which alternates between two states, one zero and the other positive. During periods when the intensity is zero, no events can occur. Smith and Karr (1983) assumed that during periods with positive intensity, events occur according to a Poisson process and that the sequence of states visited forms a Markov chain. This model is a renewal model (i.e., interarrival times are independent) and was termed the Renewal Cox process with Markovian intensity (RCM). The authors applied this model to the daily rainfall occurrences of the summer season (July to October) rainfall in the Potomac River Basin. Smith and Karr (1985b) developed statistical inference procedures and maximum likelihood methods of estimation for Cox, N–S and renewal processes. These methods can be used for direct comparison of two classes of point processes to determine which one provides a better model for a given data set.

A different type of model was proposed by Kavvas and Herd (1985). Their model is a radar–based conceptualization of ground rainfall at a location and can describe rainfall at time scales smaller than or equal to one day. Under their conceptualization, the rainfall intensity distribution over the trajectory formed while the rainfall field that passes over the ground location is used to describe the rainfall at that ground location. This model has the structure of a filtered Poisson cluster process, and its parameters are estimated from the combined radar–raingage data. The advantage of this conceptualization is that it can explicitly use the almost continuous-time radar observations, although calibration with raingage data is essential. The authors applied this model to hourly and daily time scales of rainfall in Kentucky. They used 5–min increment radar microfilm and hourly raingage data for calibration.

The advantages of the continuous–time point process models will be fully realized if they are proved to be independent of the time–scale at which the fitting of the model is done. Note, that a time–scale dependent model is still a perfectly valid model, but only for a particular scale. Also, no physical interpretation should be attached to its parameters, even if the model has a physically–based structure. For example, no inferences about the rate of the passage of fronts should be made based on the time–scale dependent parameters of the NSWN process, as some earlier studies have suggested. At present, although some models, e.g., the NSRP and the BLRP models, hold promises, more simulation studies are needed to test the performance of these models and to test the sampling properties of the estimated parameters. In general, it is understood that there is a limit to the fine–structure information that can be retrieved from the aggregate data. For example, if only daily data are available the identification of the cluster structure may be impossible, in the same way that the fine structure of a raincell cannot be possibly identified from ground measurements at distances larger than the dimension of a cell (1 – 10 km^2). In those cases the description of the process at the aggregation level of the observations may be the only meaningful approach.

Foufoula–Georgiou and Lettenmaier (1987), motivated by the statistically flexible and physically attractive structure of the point process models and the scale dependency of the continuous–time PP's (at least the ones available at the time of the study), proposed the construction of discrete–time PP models. They introduced a Markov Renewal (MR) model for the description of daily rainfall occurrences. In the MR model the sequence of times between events (defined as any wet day or hour) is formed by sampling from two geometric distributions according to transition probabilities specified by a Markov chain. The motivation behind this structure is that daily rainfall occurrences may be the result of the interaction of several rainfall generating mechanisms. For example, the first rainy day in a wet period may be the result of a frontal storm passing over a region, whereas subsequent rainy days in the same wet period may be considered secondary events. The MR process is a clustered process and has as a special case the Markov chain models. It differs from a Markov chain in the sense that the probability of having a rainy day does not depend on the condition (rain, non–rain) of the previous day, rather on the number of days since the last rain. The authors derived the statistical properties of the MR model and developed maximum likelihood estimation procedures. Combining it with a mixed exponential distribution for the amounts, they applied the model to several daily rainfall sequences in the U.S. and found it adequate, in that it preserved both the short–term and long–term (e.g., monthly totals) structure. Following this approach, Smith (1987) introduced another discrete–time PP model, namely the Markov Bernoulli (MB) process which can be viewed as a sequence of Bernoulli trials with randomized success probabilities. He derived the key statistical properties of the process, developed likelihood–based inference procedures, and applied it to daily rainfall sequences in Washington, DC. The above two discrete–time point process models, namely the MR and MB processes, are a clear advancement over the long established Markov chains and should prove useful for operational purposes, especially when only daily data are available.

2.1.2. *Space-Time Rainfall Models*

Significant ideas on space–time rainfall modeling were presented in the early 1970's by J. Amorocho and co-workers (e.g., Amorocho and Slack, 1970; Amorocho and Morgan, 1974; and later Amorocho and Wu, 1977; and Amorocho, 1981). Their model had a physically-based structure but it was of a simulation nature with no analytical capabilities; this probably hindered its further implementation and development. A major contribution in space–time rainfall modeling was the theoretical work of Gupta and Waymire (1979) and Waymire et al. (1984). They employed empirical observations of extratropical cyclonic storms and formulated a physically-based kinematic, stochastic representation of the rainfall field in space and time. Their formulation was based on the empirical evidence (e.g., Austin and Houze, 1972; Hobbs, 1978; Hobbs and Locatelli, 1978; and Houze, 1981) that precipitation patterns have a high degree of hierarchical organization and definable characteristics and behavior. In particular, Austin and Houze (1972) categorized precipitation areas according to their areal extent and lifetimes as synoptic (meso-α scale), large mesoscale (meso-β scale), small mesoscale (meso-γ scale), and raincells. Synoptic rainfall fields cover areas of the order of 10^4 km^2 and have a lifetime of one to several days; large mesoscale areas (LMSA), also called rainbands due to their elongated shape, have an extent of 10^3 – 10^4 km^2 and a lifetime of several hours; small mesoscale areas (SMSA) have extent of the order of 10^2 – 10^3 km^2 and a lifetime of approximately one hour; raincells have areal extent of the order of 1–10 km^2 and lifetimes of a few minutes to at most 1/2 hour. A systematic analysis of radar observations and raingage records for several storms (see, for example, Austin and Houze, 1972) revealed a preferred hierarchical organization of the rainfall fields, in the sense that every precipitation area at any of the above scales contained one or several of each of the smaller sized precipitation areas. For instance, the large synoptic areas (which are present with cyclonic storms, but not with frontal passage or air mass thunderstorms) usually contain several (one to six) LMSA's which appear as elongated rainbands that build, move, and dissipate within the synoptic area. LMSA's (as those within the synoptic areas and the frontal bands which themselves are LMSA's) contained several SMSA's, whereas SMSA's were rarely observed outside the LMSA's. Cells were almost always located either within the SMSA's (air mass thunderstorms being themselves SMSA's) or in a cluster (within the LMSA) forming a similar size.

Waymire et al. (1984) described in mathematical terms the above organization of extratropical cyclonic storms. In their stochastic model, the location of rainbands is specified relative to a fixed spatial origin. Multiple rainbands can occur in time and their arrival time is specified (relative to a fixed time origin) according to a homogeneous Poisson process. Each rainband (LMSA) contains circular regions, called cluster potential regions (SMSA). Upon the arrival of a rainband, the centers of the cluster potential regions occur within the rainband according to a spatial Poisson process. Once a cluster potential center is located, cells are born within it according to a three-dimensional (space–time) process. The rainbands and the cells move with constant, but different, velocities. From the above morphological description of mesoscale cyclonic precipitation fields, Waymire et al. (1984) derived the theoretical mean and covariance function of the space–time rainfall intensity at the ground. An interesting feature of that model is that it is consistent with the empirical observations of Zawadzki (1973) concerning Taylor's hypothesis. Taylor's hypothesis of turbulence (Taylor, 1938) implies, in the context of precipitation fields, that the autocovariance at some time lag, Δt at a fixed but arbitrary spatial point, is the same as the spatial covariance at a fixed but arbitrary time between two points separated by a distance Δx when space is converted to time through the constant velocity of the storm, i.e., $\Delta x = U \bullet \Delta t$. Zawadzki (1973) found that Taylor's hypothesis was valid only for periods shorter than 40 minutes; this was reproduced by the model of Waymire et al. (1984) (see also Gupta and Waymire, 1987). It should be mentioned that some of the previous models, such as the model of Bras and Rodriguez-Iturbe (1976), had assumed the validity of Taylor's hypothesis at all times.

Following these developments, Sivapalan and Wood (1987) developed a similar stochastic model to describe the space–time rainfall characteristics within a single rainband. Valdes et al. (1985) used the kinematic–stochastic model of Waymire et al. (1984) to simulate two-year traces of rainfall at a number of stations. By aggregating these sequences at different time scales they compared the performance of several temporal rainfall models (see discussion of temporal rainfall models).

Kavvas et al. (1987a) studied the stochastic description of extratropical cyclonic rainfields at the synoptic and subsynoptic scales. Their model, although of the same general structure as that of Waymire et al. (1984), differs from it in several issues. Some of these issues are: (a) the synoptic scale description which accounts for the existence and movement of several cyclonic rainbands having preferred locations, orientations, and spatial characteristics relative to their cyclone center; (b) the relaxation of some assumptions related to the hierarchical organization structure of rainfields thus permitting the cores and cells to appear not only within but also outside the rainbands and cells to appear outside the cores, etc.; and (c) the relaxation of some assumptions related to the velocity of the SMSA's and cells relative to the rainbands. More details on the differences between these models can be found in Kavvas et al. (1987a); see also the work of Kavvas et al. (1987b). At present, this model has only been used in a simulation mode with parameters in agreement with radar-based observations reported in several studies. If the model is to be used for operational hydrologic studies, analytical derivations of some of its statistical properties are needed.

A simpler model for space–time rainfall was proposed by Smith and Karr (1985a,b). This model is composed of a description of the temporal occurrence of storms (assumed a homogeneous Poisson process), a description of the spatial distribution of the raincells within a storm (assumed a spatial Poisson process), and the description of the rainfall patterns within a cell. For this five-parameter model the authors developed maximum likelihood estimation methods but commented that this "would become intractable under formulations only slightly more general".

Rodriguez-Iturbe et al. (1986) studied the spatial structure of the total rainfall depths within a stationary storm. The models they developed consist of stationary cells distributed in space according to a Poisson or an N–S process. The rainfall depth at the center of the cells is an i.i.d random variable, exponentially distributed. Rainfall is distributed within each cell by a spread function of a specified form (exponential, quadratically exponential, and linear). The rainfall depth at any point is the superposition of the depths from all the contributing cells. They showed that for the case of exponentially distributed center depth and quadratically exponential spread function, the total storm depth at any point has a Gamma distribution. This may be a useful tool in partially justifying the structure of the raincell distribution. They applied their model to data from the Upper Rio Guaire Basin in Venezuela. The model was further applied and evaluated by Eagleson et al. (1987). This later study used eight years of summer storm data from 93 stations in the 154 km^2 Walnut Gulch catchment in Arizona and compared the relative performance of the spatial Poisson process models with different spread functions for the raincell depths. They concluded that the models with the exponential and quadratic exponential spread functions were able to describe the spatial rainfall distributions of the essentially stationary air mass thunderstorms in the Southwestern U.S.

An effort to incorporate radar and raingage observations in the model development and parameter estimation was reported by Smith and Krajewski (1987). These authors, recognizing the limitations of using only ground data for space–time rainfall estimation, combined the strength of the radar data in delineating wetted areas with the accuracy of ground measurements into a single estimation technique. In their model, the structure of rainfields within a wet period consists of circular cells organized within Poisson distributed rainbands. The cells have random but constant intensity. The temporal evolution of the wet–dry periods is governed by a Markov chain. The authors developed parameter estimation methods and applied the model to daily rainfall fields in the tropical Atlantic region covered by the GATE experiment. They note that a limitation of the model is its inability to deal with low–intensity rain hiding more apparent structure of a rainfall field.

An important area of precipitation research which is not reviewed in the present article is the scaling behavior of precipitation fields and the fractal models of rain. The reader is referred to the work of Waymire (1985), Lovejoy and Schertzer (1985), Lovejoy and Mandelbrot (1985), and Schertzer and Lovejoy (1987). Georgakakos and Kavvas (1987) review the fractal rainfall models of Lovejoy and co-workers and draw some comparisons between these models and the other classes of space–time rainfall models. As these authors comment, the fractal models of Lovejoy et al. are of a simulation nature with no analytical capabilities; at present, this hinders their application to operational hydrology.

2.2. Quantitative Precipitation Forecasting

Crucial to the prediction of floods in real–time is the accurate prediction of the precipitation rate. This is especially important for the improvement of forecasts for small watersheds where the lag time between rainfall occurrence and outflow from the basin is short. A recent (1982) Program Development Plan for improving Hydrologic Services in the U.S., illustrates that 50 percent of the forecast points for communities across the U.S. have potential forecast lead times (maximum possible lead times with uniform distribution of rainfall over the basin) of less than 10 hours while 25 percent have less than 4 hours of forecast lead times (Georgakakos and Hudlow, 1984). Clearly, accurate precipitation predictions for even a few hours into the future would result in valuable increases in effective lead time. Several models and procedures for the real–time quantitative precipitation prediction exist (see Georgakakos and Hudlow, 1984, for a review). However, it was only recently that dynamic precipitation models, suitable for use in hydrologic models of real time forecasting of floods and flash floods, were presented in the literature (Georgakakos and Bras, 1984a,b).

Distinctive characteristics of the latter models are their hydrologic spatial and temporal scale, and their state–space form which allows the use of a state estimator for the automatic, real–time update of the precipitation model state from surface precipitation observations (see analysis in Georgakakos, 1986). The state estimator also gives the capability for probabilistic forecasts that quantify uncertainties due to observation sensor errors, model structure and parameter errors, and input error. The term stochastic–dynamic model in this context denotes a dynamic model complemented by a state estimator. Previous reviews of this class of models were included in the works of Georgakakos and Kavvas (1987), Georgakakos (1987), and Georgakakos and Brazil (1987). A short review of the physical aspects of the formulation of hydrologic precipitation models follows along the lines of Georgakakos (1987).

The Georgakakos and Bras (1984a,b) station precipitation model was used to generate precipitation predictions based on surface air pressure, temperature, and dew–point temperature data. The model equations can be written in the following form:

$$dx_p(t)/dt = f[\mathbf{u}(t)] + h[\mathbf{u}(t)]\ x_p(t) \tag{1}$$

and

$$y_p(t_k) = \Delta t\ \phi[u(t_k)]\ x_p(t_k), \quad k = 1, 2, \ldots \tag{2}$$

where $x_p(t)$ is the condensed water–equivalent mass (or volume) at time t in a unit area column that extends from the bottom to the top of the clouds (in kilograms per square meter or millimeters per square meter); $\mathbf{u}(t)$ is the vector of the precipitation model inputs (i.e., surface air temperature, pressure, and dew-point temperature) at time t; $\{h[\mathbf{u}(t)]\, x_p(t)\}$ is the outflow mass (or volume) rate of condensed water equivalent from the cloud column at time t (in kilograms per square meter per second or in millimeters per square meter per second); and $f[\mathbf{u}(t)]$ is the inflow mass (or volume) rate of condensed water equivalent into the cloud column at time t (in kilograms per square meter per second or in millimeters per square meter per second). Outflow is due to precipitation or local cloud–top anvil formation, while inflow is due to condensation and air mass ascent. Subcloud evaporation reduces the precipitation mass that reaches the ground to a value smaller than the one obtained at the cloud bottom level. The precipitation mass (or volume) reaching the ground is collected between time instants t_k and t_{k+1}, for $k = 1, 2, \ldots, (\Delta t = t_k + 1 - t_k)$ and is represented by $y_p(t_k)$ (in kilograms per square meter per Δt or millimeters per square meter per Δt). The instantaneous precipitation rate at ground level is given by $\{\phi[\mathbf{u}(t_k)]\, x_p(t_k)\}$ (in kilograms per square meter per second or millimeters per square meter per second).

The details of model derivation and expressions for $f(\ )$, $h(\ )$, and $\phi(\ )$ are given by Georgakakos and Bras (1984a,b). The model is based on a convective parameterization of the vertically averaged updraft velocity as a function of the surface–air meteorological model input variables:

$$v = \varepsilon_1 \sqrt{c_p}(T_m - T_s') \tag{3}$$

where ε_1 is a model parameter, proportional to the square root of the ratio of kinetic to maximum thermal energy per unit mass of ascending air; c_p is the specific heat of dry air under constant pressure; T_m is the cloud temperature at a certain pressure level p'; and T_s' is the corresponding potential ambient air temperature, determined from surface air meteorological data using adiabatic ascent of air from the ground surface. The level p' is defined by:

$$p' = p_s - (p_s - p_t)/4 \tag{4}$$

where p_s is the cloud bottom pressure computed from heat–adiabatic ascent and p_t is the cloud top pressure.

Using the cloud updraft velocity, v, the condensation inflow rate $f[\mathbf{u}(t)]$ was obtained by:

$$f[\mathbf{u}(t)] = \Delta W\, \rho_m v\, dA \tag{5}$$

where ΔW is the mass of liquid water resulting from condensation during the pseudo–adiabatic ascent of a unit mass of moist air; ρ_m is a vertically averaged (from cloud top to cloud bottom) density of moist air; and dA represents the unit area measure.

Cloud microphysics transforms the discrete mass flux of the falling precipitation drops to the continuous precipitation rate through the use of an exponential particle size distribution:

$$n(D) = N_o \exp(-D/\varepsilon_2) \tag{6}$$

where N_o is a parameter expressed as a function of the model state variable x_p; D is the drop diameter; and ε_2 is a free model parameter representing the average diameter of hydrometeors in the cloud column.

Since the emphasis on flood, especially flash flood, prediction is in areas of $10^2 - 10^3$ km^2 the use of the station precipitation model requires surface meteorological data sampled a points separated by distances of similar spatial scale. On the average, the surface meteorological data stations in the U.S. have separation distances greater than 100 km. An interpolation procedure is thus necessary to provide input to the precipitation model for all the possible locations of interest. Georgakakos (1986) developed and tested such an interpolation procedure. The variables interpolated were surface air temperature, pressure, and dew-point temperature. The interpolation accounts for altitude-varying terrain. For flat terrain the procedure reduces to a linear spatial interpolation (weights inversely proportional to distance). Average interpolation errors of about 1°K were obtained from tests in Oklahoma and about 2°K from tests in Montana for the surface air temperature variables. The pressure-variable errors were about 1 mbar. Station distances were greater than 150 km in the test cases. Complemented by the input interpolation procedure, the precipitation model can be used to predict mean areal precipitation for any drainage basin, given only surface air meteorological data at a few stations.

Several flash floods are the result of orographic enhancement of precipitation followed by the resultant fast-moving flood waves on steep slopes. The previously described spatial interpolation methodology can produce estimates of surface air meteorological variables in mountainous terrain. Through the terrain variation of the aforementioned meteorological variables that are used as an input to the precipitation prediction model, the orographic effects are partially taken into account. However, if accurate prediction of precipitation in mountainous areas is to be obtained, the most important effect of orography, namely, the enhancement of the updraft velocity and of the mass influx into the clouds, should also be simulated by the model.

Several studies have examined the issues concerning the modeling of orographic enhancement, given operationally available meteorological data (e.g., Elliott and Shaffer, 1962; Bell, 1978; Rhea, 1978). The basic idea is that the updraft velocity enhancement which is due to orography is equal to the inner product of the horizontal wind vector $\vec{W}$ and the local topographic gradient vector, $\Delta\vec{H}$,

$$V_o = \vec{W} \, \Delta\vec{H} \tag{7}$$

where V_o is the magnitude of the orographic component of the updraft velocity. The operational models presented in the literature to date vary in the way they treat the vertical distribution of the updraft velocity enhancement and the conditions under which enhancement can occur.

In an ongoing research effort, the stochastic–dynamic model discussed above has been modified to incorporate orographic enhancement of the updraft velocity. When only surface meteorological data are available (e.g., in cases when the forecast lead time is less than 6–hours), Equation (7) is used to estimate the surface updraft enhancement and a linear reduction of the orographic updraft velocity component with height is assumed with the vanishing level at the cloud top level. No orographic enhancement is computed in cases when the horizontal surface wind is less than 2.5 m/sec or when the surface relative humidity is less than 65 percent. Thus, given that the aforementioned criteria for the existence of orographic enhancement are fullfilled, the vertically averaged updraft velocity (see Equation (3)) is incremented by the amount of $V_o/2$, while the condensation input rate at cloud bottom (see Equation (5)) is incremented by the amount $\Delta W \, \rho_m \, V_o \, dA$.

The problem of spatial interpolation of the wind vector arises in cases where orographic enhancement is significant. In the present developments, the wind vector observed at an up-wind station is assumed to be representative of the wind at the drainage basin of interest. In cases where the drainage basin is divided into more than one orographic zone, a precipitation prediction model (of the type previously described) can be formulated for each orographic zone. Then the drainage basin mean area precipitation at time $t_k[\bar{y}_p(t_k)]$ can be computed by:

$$\bar{y}_p(t_k) = \sum_{i=1}^{L} \frac{A_i}{A} \; y_{p_i}(t_k) \tag{8}$$

where A_i is the area of the i–th orographic zone; A is the total area of the drainage basin; and $y_{pi}(t_k)$ is the precipitation prediction at time t_k computed for the i–th orographic zone. The areas of the L orographic zones satisfy:

$$\sum_{i=1}^{L} A_i = A \tag{9}$$

Previous results obtained by the stochastic dynamic precipitation model (see Georgakakos and Bras, 1984a,b: and Georgakakos, 1984) appear encouraging in that the prediction residuals have smaller variance than the precipitation observations while at the same time physically realistic values were obtained for model parameters such as updraft velocity, cloud–averaged drop diameter, and cloud top pressure. Experience in using the model with data from various storm types points to some model weaknesses which should be addessed if improved model predictions are to be obtained. One of the characteristics of the model predictions (especially the deterministic predictions) is the under–estimation of excessive rainfall rates. Several model assumptions contribute to such an underestimation. The assumptions we consider to be the most important in that respect are:

(1) The updraft and rainfall occur uniformly in the space of the column of the precipitation model. In actual storms, regions of updraft are separate from regions of downdraft where drops fall with respect to the ground surface.

(2) The terminal group velocity of falling drops during intense rainfall might be significantly different from the terminal velocity of isolated drops which is what the model uses. The model would tend to underestimate not only high rainfall intensities but also rainfall mass since evaporation in the subcloud layer would be acting for shorter time intervals when group velocity is considered.

(3) Mass advection from the sides of the column is not taken into consideration. A two–dimensional representation is needed.

In an ongoing research effort (e.g., Georgakakos and Lee, 1987), the model formulation has been modified to include a two–dimensional representation, a group velocity component, and a separation of updraft and downdraft regions in each computational grid square. Sensitivity analysis of various, mainly convective, storms is underway. Preliminary results indicate improvement in the performance of the hourly predictions of the deterministic model.

3. Conclusions

The last decade has witnessed significant advancements in the areas of space–time precipitation modeling and forecasting. In terms of stochastic precipitation models, it seems that the present modeling developments outperform our ability to use them in an operational framework. The missing link is reliable parameter estimation. For example, the Waymire et al. (1984) model has fourteen parameters to be estimated. As expected, identification problems may arise when all of these parameters are attempted to be estimated from the time–aggregated data at a sparse raingage network. It is desirable that some of these parameters (such as the size of the cells and the parameters of the rainfall distribution within the cells, etc.) and also some of the model components (such as the functional form of the spread function within the cells) are estimated from data at very fine spatial scales or are physically inferred. This approach would also guarantee the physical meaning of the parameters, whereas parameters estimated from aggregated data and based on hypothesized (and not data-identified) model components should be viewed with caution. For example, the Eagleson et al. (1987) study seems to favor the quadratic exponential spread function, while studies of cell properties based on radar data (e.g., Konrad, 1978; Drufuca, 1977) seem to indicate that the exponential decay function may be more appropriate. Although both models may give similar results in terms of describing the overall spatial rainfall structure, the parameters of the model may change significantly (e.g., compare the values of Table 3 of Rodriguez-Iturbe et al., 1987b); therefore, the physical meaning of the model parameters may be questionable.

It may be that better estimation of stochastic model parameters requires employment of radar or satellite data in conjunction with ground observations. However, the simultaneous use of multiple sensors is not yet developed to the point that they can be used for estimation purposes. One of the main problems is the different properties of the errors of each one of those sensors (see Krajewski, 1987; Krajewski and Georgakakos, 1985).

Concerning the developed stochastic-dynamic quantitative precipitation prediction models, several improvements are feasible in the near future. Some of these improvements are:

(1) Incorporation of data from remote sensors within the state space formulation of the models. A major challenge in this area is the characterization of the errors in data from remote sensors.

(2) Refinement of the physical basis of the models. This aspect would benefit from the training of hydrologists in the area of hydrometeorology.

(3) Coupling of the precipitation models with the large-scale Numerical Weather Prediction Models (NWPM). As a first step, use of NWPM forecasts as the precipitation model input in real time, and assessment of performance in both deterministic and probabilistic predictions can be accomplished.

Although one can argue that operational hydrology needs simpler models, it should be acknowledged that further understanding of the underlying space-time rainfall structure cannot be gained unless physically realistic rainfall models are extensively studied and tested with good quality, high resolution space-time data. Until these models are fully tested, however, the hydrologist needs to exercise judgement to best use the scientific developments and to select a model both appropriate for the modeling objectives and consistent with the available data.

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