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Do the current landscape evolution models show self-organized criticality?

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Abstract. In this note we question the relevance of the self-organized criticality concept as currently applied to landscape evolution modeling. We also express concerns about possible inference problems in testing hypotheses about landscape multifractality using simulated landscapes.

1. Introduction

Landscape and river network evolution modeling is an active area of research [e.g., *Willgoose et al.*, 1991a, b; *Howard*, 1994; *Kramer and Marder*, 1992; *Leheny and Nagel*, 1993; *Takayasu and Inaoka*, 1992; *Rinaldo et al.*, 1993; *Rigon et al.*, 1994; *Rodriguez-Iturbe et al.*, 1994]. In this article we concentrate on the work of *Takayasu and Inaoka* [1992] and *Rinaldo et al.* [1993] (see also *Rigon et al.* [1994] and *Rodriguez-Iturbe et al.* [1994]), who presented models for river networks and landscape evolution and claimed that their behavior shows self-organized criticality (SOC). We argue that these models do not really fall under the SOC framework, since none of their states behaves as a critical state. On a secondary but related account, we give some thought to the recent hypothesis that multifractality in landscapes might be the result of heterogeneities of field properties [*Rodriguez-Iturbe et al.*, 1994]. Although this hypothesis seems reasonable and may well be true, the way it was tested using simulated landscapes from an "SOC" model has potential inference problems and warrants further investigation. The implications of our arguments may be significant for further development of landscape evolution models and interpretation of the underlying mechanisms of scaling observed in natural landscapes.

2. What Is a Critical State in Traditional Systems and in Systems Showing Self-Organized Criticality?

Near critical points (i.e., at limiting states of equilibrium of two-phase systems when the phases become identical) and points of second-order phase transitions, physical systems show anomalies in both static properties (thermodynamic coefficients, correlation length) and dynamic properties (relaxation rates, transport coefficients). These anomalies are called critical phenomena [see, e.g., *Ma*, 1976; *Patashinskii and Pokrovskii*, 1979], and the state of the system is called the critical state. It is established in modern theories of critical phenomena that large-scale fluctuations play a crucial role in the behavior of systems in the vicinity of the critical state. The correlation length of fluctuations (roughly their average length) grows infinitely as the system approaches the critical state [see, e.g., *Ma*, 1976, chap. 3; *Patashinskii and Pokrovskii*, 1979, chap. 2]. This means that any part of the system in the critical state can "feel" changes in other parts (cooperative behavior). Therefore if the system is in the critical state, a small local perturbation can cause a significant change in the configuration of the whole system. In the critical state, systems show both static and dynamic scaling. These two phenomena, expressed by scaling in the distribution of correlation lengths of fluctuations (implying lack of characteristic scale in space) and by corresponding power law distribution in relaxation times of the fluctuations (absence of temporal scale), respectively, are fundamentally related. Qualitatively, the reason for this relation is simple: the longer the correlation length of a fluctuation, the longer the time it needs to relax. Specifically, the characteristic frequency of fluctuations depends on their characteristic length as a power law [see, e.g., Hohenberg and *Halperin*, 1977; *Ma*, 1976; *Patashinskii and Pokrovskii*, 1979].

The concept of self-organized criticality was introduced by *Bak et al.* [1987] as a general organizing principle governing the evolution of nonlinear systems to a state which exhibits the features of a traditional critical state: there is no natural scale in this state, and the systems fluctuate strongly in space and time, exhibiting spatial and temporal scaling. This enabled Bak et al. (by analogy to traditional critical phenomena) to call the state reached by such systems critical and to coin the term "self-organized criticality" for these phenomena. For instance, in the typical example of a sand pile, as the pile is built up, the characteristic size of the largest avalanches grows until the pile reaches the critical state. As soon as this happens, one sand grain can produce an avalanche of any size up to the size of the system [*Bak et al.* 1987, p. 382; 1988, p. 365]. The strength of the avalanches (number of particles involved in an avalanche) follows a power law distribution. Also at the critical state, the sand pile surface shows fractal geometry. Another example of a system showing self-organized criticality is presented in a model of earthquakes [see, e.g., *Bak et al.*, 1994]. In this model, the transfer (according to some rule) of force to the neighboring elements may cause them to become unstable, thus triggering a chain reaction (modeling the earthquake). Again, this system evolves to a SOC state which is characterized by the presence of earthquakes of all possible energies, the energy of the earthquakes being distributed as a power law.

In systems showing self-organized criticality, similarly to traditional critical phenomena, temporal and spatial behavior are interrelated. In fact, *Bak et al.* [1987, 1988] show that there is a close connection between the "$1/f$" noise observed in many natural phenomena and the spatial self-similar fractal structure of the critical state. The relationship between spatial fractal

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behavior and temporal scaling in SOC systems was further studied and formally established by *Maslov et al.* [1994].

Systems showing self-organized criticality are characterized by two seemingly contradictory but in essence complementary features:

1. On the one hand, large, catastrophic events are intrinsic, unavoidable features of a system at the critical state [*Bak et al.*, 1994, p. 69]. Under small perturbations, such systems are proven to undergo changes which show power law distribution of strength. These changes can alter the configuration of the system dramatically. This feature corresponds to the dynamic properties of traditional critical states.

2. On the other hand, however, once the critical state is reached, the system stays there [*Bak et al.*, 1988, p. 365]. In other words, for systems showing SOC the critical state is an attractor of the dynamics. This means that the state of self-organized criticality is stationary; once it is achieved, the statistical properties of the system, such as spatial scaling and the power law frequency distribution of the events that change the system (avalanches or earthquakes), do not change. Except for fluctuations (which can be very strong, though; see feature 1), the system does not evolve. In that sense, the critical state of such systems is their destination point. The interplay of these two features determines the nontrivial behavior of systems exhibiting self-organized criticality.

3. Do the Current Models of River Network and Landscape Evolution Show Self-Organized Criticality?

In the model of river networks and landscape evolution introduced by *Rinaldo et al.* [1993] and further investigated by *Rigon et al.* [1994] and *Rodriguez-Iturbe et al.* [1994], self-organization takes place. However, we argue that none of the states of the model is critical and that the model therefore cannot be called an SOC model. In brief, the evolution rules of the model are as follows:

1. A given threshold shear stress value τ_c is assigned to the model.

2. Two variables are assigned to each site of a two-dimensional lattice: an elevation z_i and a discharge surrogated by the draining area A_i.

3. For each site the shear stress is computed as $\tau_i \propto A_i^{0.5} \Delta z_i$, Δz_i being the drop along the steepest descent. The computed τ_i are compared with the assigned critical shear stress τ_c. The elevation of the site j having the maximum exceedance of τ_j over τ_c is then reduced to the value which yields $\tau_j = \tau_c$ (this reduction simulates erosion). The released "mass" is evacuated from the system.

4. Drainage directions (fixed by the steepest descent in each site) are recomputed because they are altered as a result of the modified elevation of the site j. Accordingly, the values of the draining areas A_i and shear stresses τ_i are recalculated too.

5. Steps 3 and 4 are repeated until there are no exceedances of τ_i over τ_c.

6. The state achieved after step 5 is perturbed at random by adding elevation to a node. The perturbation may lead to a readjustment of the structure, and this is repeated until further perturbations do not induce variations in the configuration of the system.

For more details, the reader is referred to *Rinaldo et al.* [1993, p. 824]. The earlier model of *Takayasu and Inaoka* [1992] in many respects is analogous to the model described above except that the erosion is not modeled as a threshold process.

The authors of the model call the intermediate states reached after every step 5 (when there are no exceedances of τ_i over τ_c) critical. For example, *Rigon et al.* [1994, p. 976] describe step 5 of the model as follows: "5. Steps 3 and 4 are repeated until no exceedances are isolated. Thus at any stage the studied system evolves to a critical state (step 5). 6. The critical state 5 is perturbed . . . ." The claim that the state of the system after each step 5 is critical, is also made by *Rinaldo et al.* [1993, p. 824] and *Rodriguez-Iturbe et al.* [1994, p. 3532]. We argue that these states cannot be considered critical for two reasons. First, the states achieved after every step 5 are intermediate; the statistics of the system continue to change systematically from step to step, up to the final configuration. This implies that the system does not exhibit one of the two inherent features of SOC systems, namely, feature 2. As was mentioned earlier, this feature requires that in SOC systems once a critical state is reached, the system stays there; i.e., a system showing self-organized criticality is stationary in the critical state, which is an attractor of the dynamics of the system. Second, the configuration of the system after every step 5, and before it reaches its final state, does not yet show fractal structure; it still has a characteristic scale as clearly demonstrated in Figures 4a and 4b of *Rinaldo et al.* [1993, p. 824], where one sees that a power law distribution of drainage areas and stream lengths is not present at that state. This again indicates that the state of the system after every step 5 is not a critical state.

Fractal structure indicated by a power law distribution of spatial characteristics (drainage areas and stream lengths), is present in the final state of the model. The authors of the model do not explicitly call the final state critical (although at one point they mention that "the studied system always evolves into a stable critical state" [*Rinaldo et al.*, 1993, p. 822]). However, as was discussed earlier, they do say that the critical state is achieved after each step 5. If this were true, then the final configuration of the system achieved after a series of perturbations followed by step 5 would also have to be critical, since once the critical state is reached, the SOC system remains there. We point out, however, that although the power law relations in spatial characteristics could mislead someone to consider this final state critical, one must note that SOC systems in the critical state also show a power law distribution of changes under small perturbations (e.g., a power law distribution of avalanche sizes or earthquake strengths), described earlier in the discussion of feature 1. In contrast to the behavior of SOC systems, in the final state of the landscape evolution model there are simply no changes at all, let alone the absence of large catastrophic events (characteristic of a critical state and implied by a power law distribution of changes). Indeed, since the drainage pattern (and consequently the feeding areas) do not change in the final state of the model, the shear stresses do not change either, and thus they do not exceed the critical value, which is a necessary condition for erosion to occur in this model (step 4). The analogous model of river network and landscape evolution by *Takayasu and Inaoka* [1992] also evolves into a state in which the river patterns are frozen. It should be pointed out, however, that in the final state of this model, the water flow continues to erode the surface. Takayasu and Inaoka recognized that the final state of their model "is very different from that of the SOC model" [*Takayasu and Inaoka*, 1992, p. 966] and introduced the term "a new type of SOC system" to imply a system with spatial but not temporal scaling. We argue that this terminology is misleading and that not any system evolving to a fractal structure can be called an SOC system. As was mentioned earlier, in the critical state, both for traditional and for SOC systems, spatial and dynamic scaling are fundamentally related. Therefore a claim that a system exhibits SOC because it evolves into a state characterized by fractal geometry, i.e., shows "critical behavior in space" but does not show any changes under perturbations, seems internally contradictive. Thus since the final configuration of the system, i.e., the drainage pattern in the model of *Takayasu and Inaoka* [1992] and the drainage pattern and landscape itself in the model of *Rinaldo et al.* [1993], does not change under perturbations, the final state of the system cannot be considered critical.

In a recent popularized article by *Bak and Paczuski* [1993], the model of *Rinaldo et al.* [1993] was cited as an example of SOC models. To the general scientific audience, this in itself could put to rest any reservations such as the ones we raise here. However, we invite the reader to carefully examine this article, which in fact further supports rather than weakens our arguments. For example, *Bak and Paczuski* [1993, p. 39] stress that SOC systems are highly dynamic:

> A common feature of these complex systems is that they are driven by slowly pumping in energy, which is stored and later dissipated ..., in an avalanche process ... They exist in metastable states and small, seemingly insignificant, increments in energy input can trigger an arbitrary large avalanche."

This is completely different from (in fact, opposite to) the behavior of the considered landscape model which evolves into a state where perturbations do not change the configuration of the system. Furthermore, *Bak and Paczuski* [1993, p. 40] acknowledge that traditional equilibrium systems in a critical state also exhibit "fluctuations of all sizes and durations," which again implies not only spatial, but also temporal variability in such systems. Thus calling the considered landscape model a SOC model contradicts not only the relatively new concept of SOC systems, but also the concept of a critical state established in equilibrium physics long ago.

4. Caution for Spurious Multifractality

Rodriguez-Iturbe et al. [1994] tried to reproduce with their landscape evolution model the observed multifractal (as opposed to monofractal) structure of natural landscapes and argued that "multiple-scaling behavior ... cannot be explained in terms of homogeneous processes" and that "heterogeneity of field properties is needed for multiple scaling to emerge" [*Rodriguez-Iturbe et al.*, 1994, p. 3538]. To test this hypothesis, they introduced spatial inhomogeneity in the critical shear stress τ_c in two different ways. First, the spatial variability of shear stress was introduced as a correlated random field (lognormal field $\tau_c(\mathbf{x})$ with $\langle\tau_c\rangle = 1$, $\sigma_\tau^2 = 0.2$, and an exponential correlation structure with an integral scale equal to 2 pixels). In this case the simulated landscapes showed monofractal behavior [see *Rodriguez-Iturbe et al.*, 1994, Figure 9]. Second, inhomogeneity was introduced in a different way: a "highly bimodal distribution of critical shear stress" was used with values $\tau_c = 2$ for the upper half of the field divided along the diagonal through the outlet, and $\tau_c = 0.5$ for the lower half. The authors state that the elevation field [*Rodriguez-Iturbe et al.*, 1994, Plate 2] shows multiscaling behavior in this case, as is

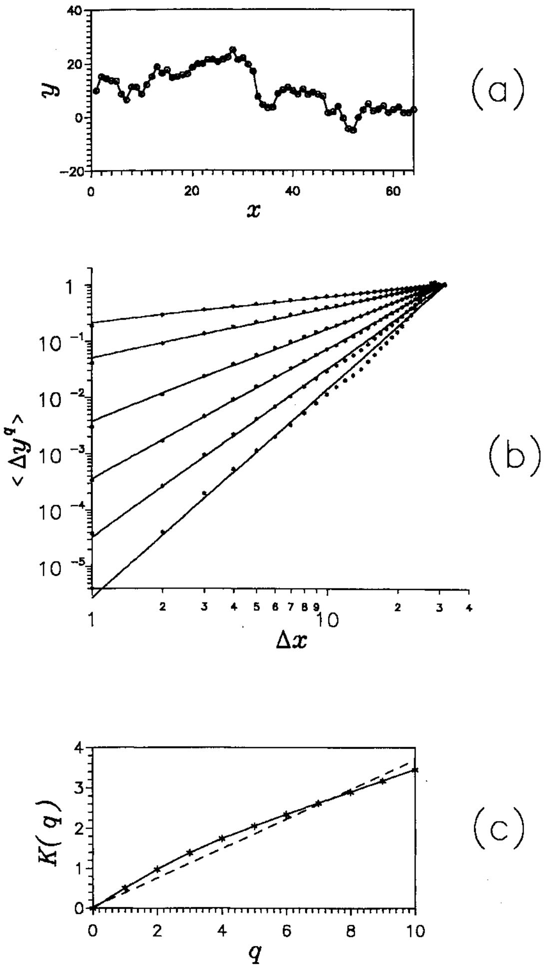

Figure 1. (a) A realization of simple Brownian motion with a step (in the middle) imposed. (b) Behavior of qth moments of the process shown in Figure 1a (the moments are produced by averaging over 500 realizations). Deviations from scaling produced by the imposed step are deceptively small, so the dependencies can easily be taken for straight lines; from bottom to top, the lines are for q = 0, 2, 4, 6, 8, 10. (c) $K(q)$ curve produced from Figure 1b under the (erroneous) assumption that Figure 1b shows scaling. This example depicts the possibility of inferring multifractality for a process which is clearly monofractal but has an imposed step (change in level).

implied from the nonlinear behavior of the $K(q)$ function [see *Rodriguez-Iturbe et al.*, 1994, Figure 10]. However, one can see in Plate 2 that the introduction of inhomogeneity of the second type produced a significant step in the landscape along the line dividing the two zones. We demonstrate that imposing such a step onto a monofractal object, without any other change in the structure of the object, can lead to a deviation of its behavior from scaling which can be misinterpreted as multifractality.

The curve shown in our Figure 1a is a realization of simple Brownian motion with a step imposed: $x(t) = B(t) + a$ for $0 < x \le 32$ and $x(t) = B(t)$ for $33 \le x < 64$ (same length as in the work by *Rodriguez-Iturbe et al.* [1994]), with $a = 10$. Following the representation of *Rodriguez-Iturbe et al.* [1994, Figure 12] the points in our Figure 1b, i.e., moments $\langle\Delta y^q\rangle$ versus distance $\langle\Delta x\rangle$ for different powers q, were calculated. The points show some deviation from straight lines, but the deviation is not very strong, making it easy to mistake this behavior for scaling (especially given the statistical variability in the moment estimates). In this case, one is bound to get misleading results if the calculation of the $K(q)$ curve is done using the "slopes" of the "straight lines" in Figure 1b. Indeed, though the object we analyzed is definitely not a multifractal, the curve $K(q)$ obtained in this way shows a deviation from linearity, as is shown in Figure 1c. Of course, the $K(q)$ curve obtained for the same Brownian motion without the step was perfectly linear. One can see that the curves $K(q)$ in our Figure 1c and in Figure 12 of *Rodriguez-Iturbe et al.* [1994] are practically identical. Thus we caution that the deviation of the $K(q)$ curve from linearity in their Figure 12 may be caused by the step in the landscape produced by the bimodality of the critical shear-stress function and does not necessarily demonstrate the multifractal structure of the simulated landscape. Notably, the first way of introducing spatial heterogeneity in the critical shear stress τ_c, such that the heterogeneity was spread over the lattice, did not lead to a deviation of the $K(q)$ curve from linearity [see *Rodriguez-Iturbe et al.*, 1994, Figure 9].

This example demonstrates that there are potential problems one should be aware of when analyzing real and simulated landscapes. These problems deserve special study.

5. Concluding Remarks

In this note we argue that in the current river network and landscape evolution models presented by *Takayasu and Inaoka* [1992] and *Rinaldo et al.* [1993], neither intermediate states nor the final state can be considered critical, and therefore in our opinion these models cannot be attributed to models showing self-organized criticality. Although they may still be valid simulation models, we caution that some claims based on the conclusion that they are SOC models should be revised. In particular, the claim that "optimal channel networks (OCN) obtained by minimizing the local and global rates of energy expenditure" [*Rodriguez-Iturbe et al.*, 1992, 1994; *Rinaldo et al.*, 1992] "are a particular case of self-organized critical structures" [*Rodriguez-Iturbe et al.*, 1994, p. 3531] is not justified by the model. The same caution applies to a more general question raised by Rodriguez-Iturbe that all self-organized structures might evolve through some global principle of energy minimization [see *Yam*, 1994, p. 26]. This conjecture may very well be true. We just point out that it does not follow from the models under consideration.

As the originators of the SOC concept have argued, the SOC mechanism can be responsible for many natural phenomena exhibiting both spatial and temporal scaling over a wide range of scales. Therefore the SOC concept looks promising for the description of natural landscapes which are known to show spatial scaling and undergo changes constantly from small to very large scales. However, in our opinion the considered models of landscape evolution do not really show SOC behavior.

As a final remark we point out that the considered models of landscape evolution lead to a final state in which the drainage pattern (model of *Takayasu and Inaoka* [1992]) or both the drainage pattern and landscape (model of *Rinaldo et al.* [1993]) remain frozen. We question how realistic conceptually such landscape evolution models are, since natural landscapes do change constantly and never really reach a "frozen state" of equilibrium. This, however, is outside the scope of this note and calls for careful investigation in itself.

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