

Precipitation Research at UMN: Multiscale variability and Uncertainty in Rainfall Estimation and Retrieval

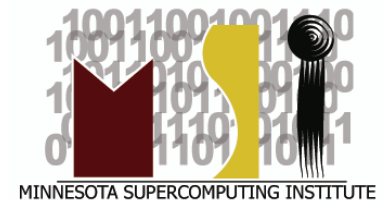
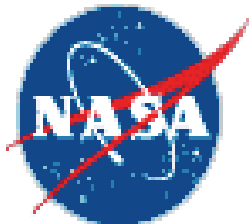
- (1) Merging multiscale observations, (2) Verification of cloud resolving models and (3) new ideas for short-time scale rainfall estimation and retrieval

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GPM meeting – Dec 2005



THREE AREAS OF FOCUS

- **Developing nonparametric schemes for merging multisensor precipitation products**

Gupta, R., V. Venugopal and E. Foufoula-Georgiou, A methodology for merging multisensor precipitation estimates based on expectation-maximization and scale recursive estimation, in press, *J. Geophysical Research*, 2005.

- **3D modeled cloud structure verification for satellite rainfall retrieval assessment**

Smedsmo, J., V. Venugopal, E. Foufoula-Georgiou, K. Droegmeier, and F. Kong, On the Vertical Structure of Modeled and Observed Clouds: Insights for Rainfall Retrieval and Microphysical Parameterization, in press, *J. Applied Meteorology*, 2005.

- **Fine-scale structure of rainfall in relation to simultaneous measurements of temperature and pressure**

Lashermes, B., E. Foufoula-Georgiou and S. Roux, Air pressure, temperature and rainfall: Insights from a joint multifractal analysis, AGU Fall Session, San Francisco, December 2005. -- add ref of Phys Letters -- Label 1,2,3

DEVELOPING NONPARAMETRIC SCHEMES FOR MERGING MULTISENSOR PRECIPITATION PRODUCTS

Issues:

- Precipitation exhibits scale-dependent variability
- Precipitation measurements are often available at different scales and have sensor-dependent observational uncertainties (e.g. raingauges, radars, and satellites)
- It is not straightforward to optimally merge such measurements in order to obtain gridded estimates, and their estimation uncertainty, at any desired scale

Objective:

To develop a methodology that can:

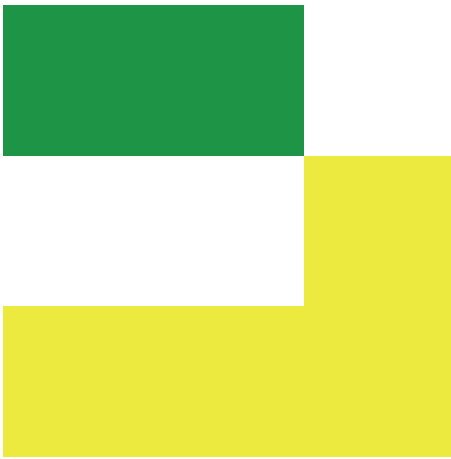
- handle disparate (in scale) measurement sources
- account for observational uncertainty associated with each sensor
- incorporate a multiscale model (theoretical or empirical) which captures the scale to scale characteristics of the process

Expected Result:

- High quality merged/blended global precipitation estimates and their error statistics at different spatio-temporal scales

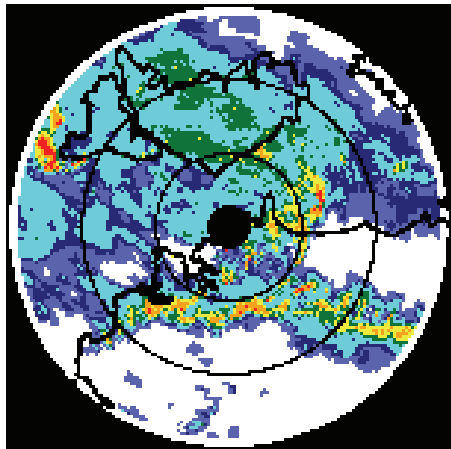
Satellite

Scale λ_1



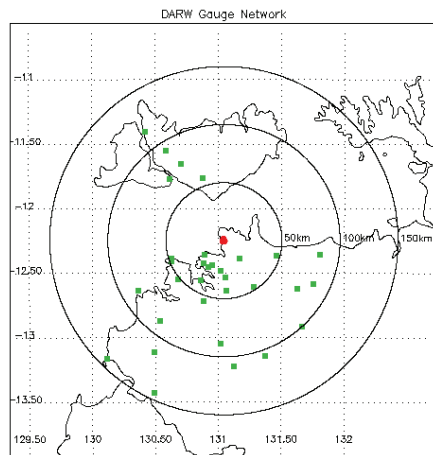
Radar

Scale λ_2

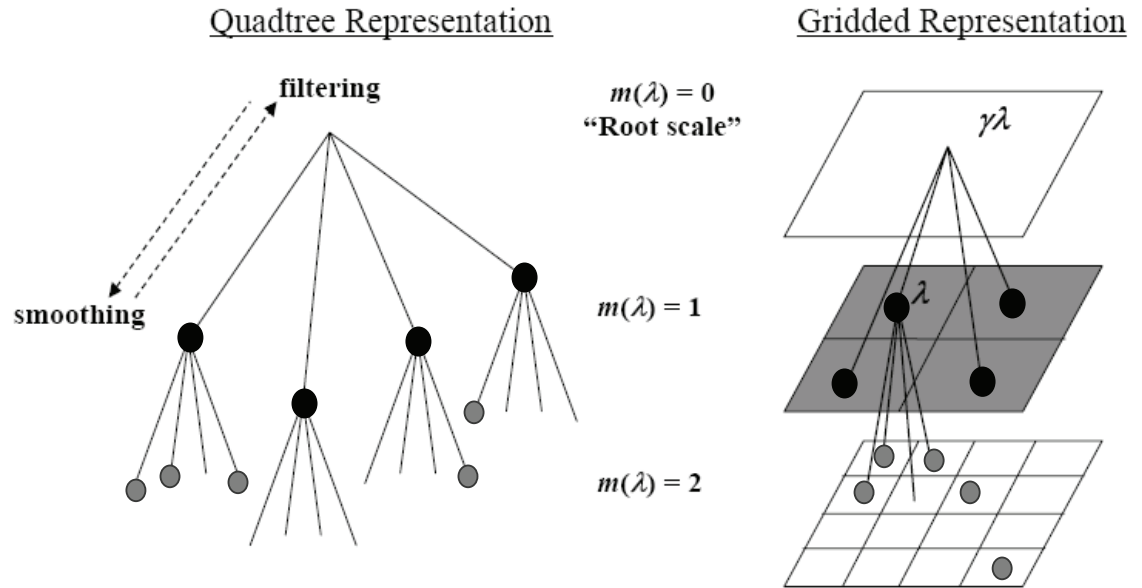


Rain gauge Network

Scale λ_3



Quadtree Representation of a Multiscale Process

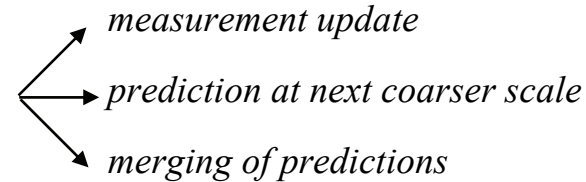


Sparsely distributed measurements at the finer scale (gray dots), and measurements at a coarser scale (solid black dots), are placed on an inverted quadtree and merged via filtering and smoothing to obtain estimates at multiple scales

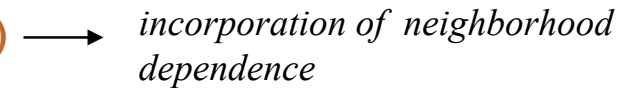
Scale Recursive Estimation (SRE)

- Consists of two steps

- **Upward Sweep (filtering)**



- **Downward Sweep (smoothing)**



- Gives precipitation estimates at multiple scales + their variance (uncertainty)
- Needs a multiscale model which relates the process from one scale to another

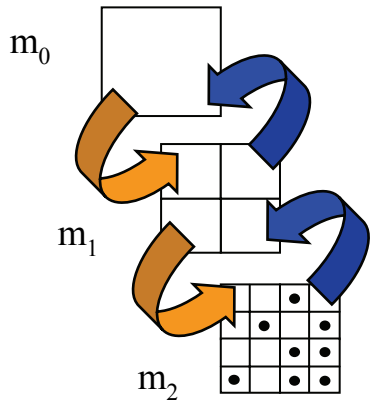


Fig. 2

State-space equation: relates the process state from one scale to another

$$X(\lambda) = A(\lambda)X(\gamma\lambda) + B(\lambda)W(\lambda), \quad W(\lambda) \sim N(0,1)$$

Measurement equation: relates process measurements and state

$$Y(\lambda) = C(\lambda)X(\lambda) + V(\lambda),$$

The parameter set of the multiscale state-space model is

$$A(\lambda), B(\lambda), C(\lambda), R(\lambda) \mid \lambda \in T, T \text{ is the set of all nodes in the tree}$$

$$A(\lambda) = 1; \quad C(\lambda) = 1; \quad \forall \lambda \in T$$

$R(\lambda)$ = Measurement error variance
associated with each sensor

$B(\lambda), \forall \lambda \in T$, needs to be estimated

Estimation of Parameter $B(\lambda)$

- **Parametric Approach**: Multiplicative Cascade models $X(\lambda) = X(\gamma\lambda) W(\lambda)$

Zero intermittency observed in precipitation presents a problem

- **Non-Parametric Approach**: Variance Reduction Curves (VRC):

Empirical variance as a function of scale

$$B(\lambda) = \sqrt{\{\text{Var}(X(\lambda)) - \text{Var}(X(\gamma\lambda))\}}$$

Sparsity of high-resolution data makes it difficult to estimate VRC

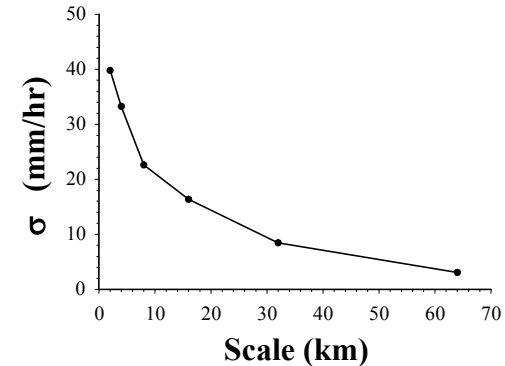


Fig. 3

Proposed solution

An approach based on Expectation Maximization which does not assume any a priori model prescribing multiscale variability, but rather uses the observations at multiple scales to dynamically evolve and simultaneously identify the parameters of the state-space equation (i.e., $B(\lambda)$).

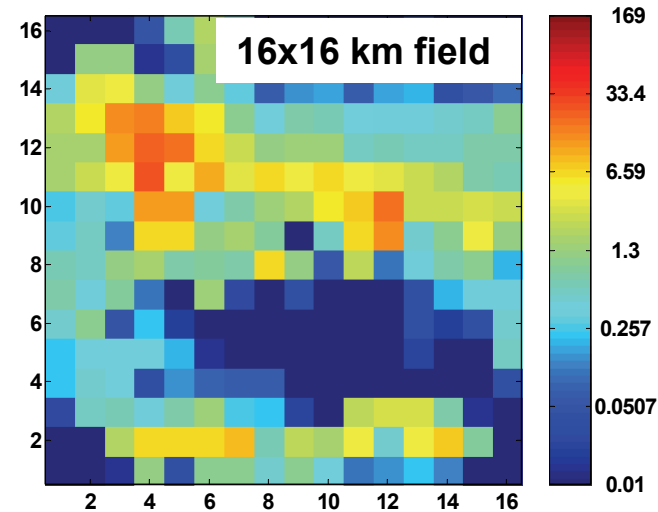
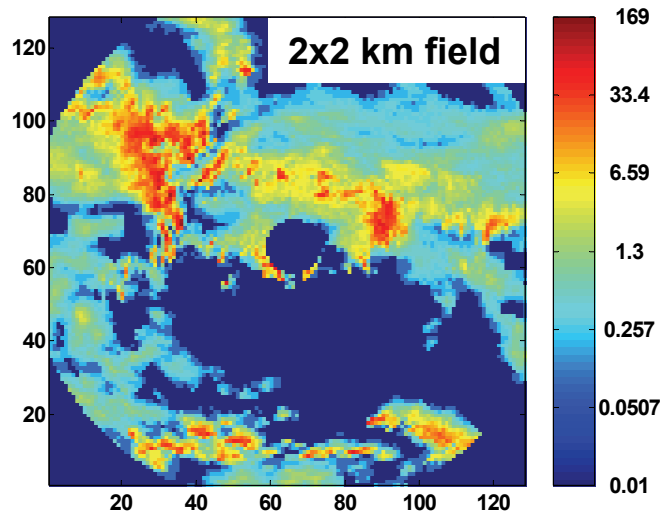
EM algorithm-based system identification

- EM algorithm is used to iteratively solve for the Maximum Likelihood (ML) estimates of the parameters of the multiscale state-space equation.
- Each iteration of the EM algorithm consists of two steps:
 - ✓ *Expectation Step (E-Step) – uses Rauch-Tung-Striebel (RTS) algorithm to estimate the state of the process*
 - ✓ *Maximization Step (M-Step) – this step re-estimates the parameters using these uncertain state estimates*

Main advantages of EM approach to SRE

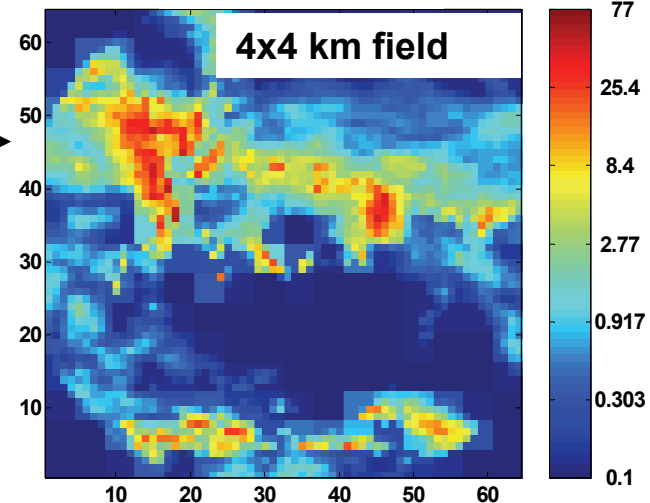
- No need for an a priori state-space model
- No need to work in log-space
- Scale-to-scale covariance explicitly used in model identification
- Observational uncertainty explicitly used in model identification

Illustration of the application of SRE-EM methodology for the purpose of blending multi-sensor precipitation estimates



SRE

ESTIMATED



Input: 2x2 km field + its uncertainty
16x16 km field + its uncertainty

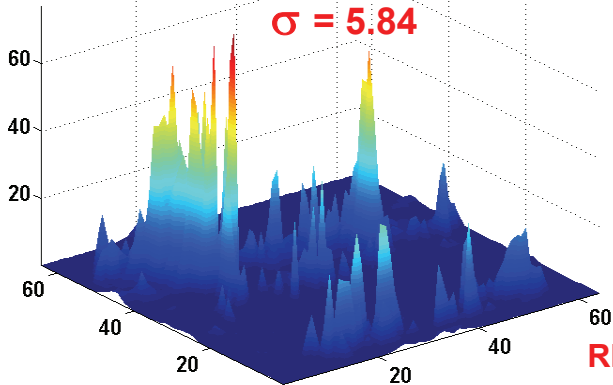
Output: 4x4 km field, along with its uncertainty.

Fig. 4

Comparison of observed vs. estimated fields

Actual Field at 4x4 km

$\mu = 1.96$
 $\sigma = 5.84$

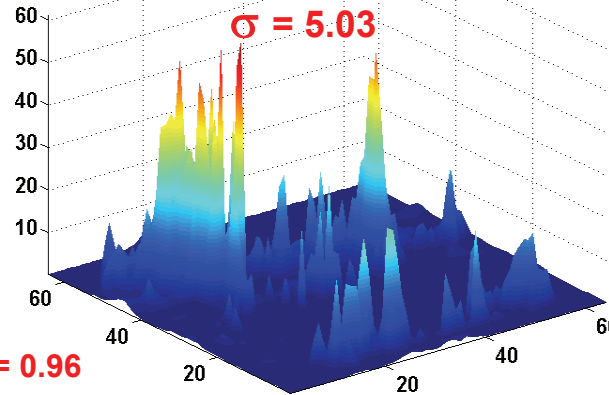


RMSE = 0.96

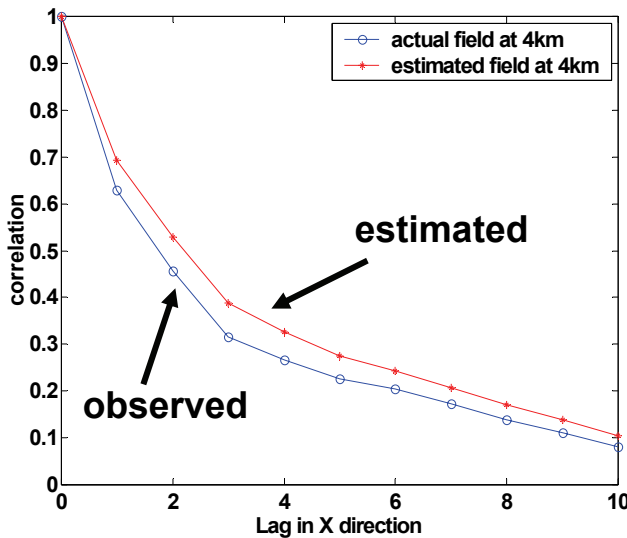
Bias ~ 0.00

Estimated Field at 4x4 km

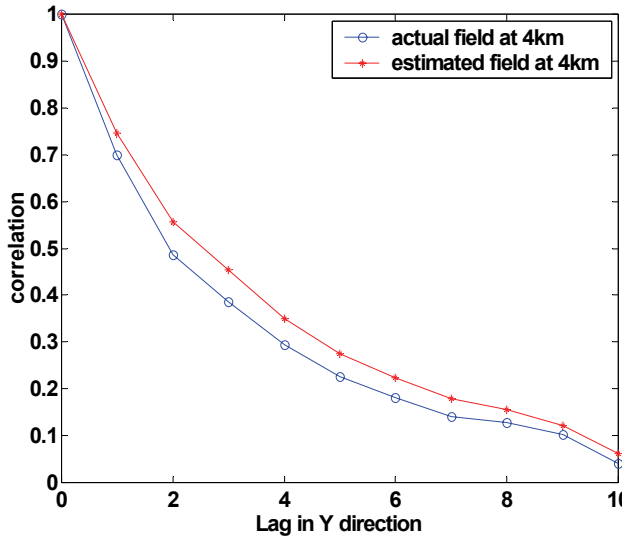
$\mu = 1.96$
 $\sigma = 5.03$



Spatial Correlation – X direction



Spatial Correlation – Y direction



- Visually, the estimated field compares well with the observed field.
- Comparison of basic statistics (mean and standard deviation) shows that SRE-EM is effective in producing unbiased estimates.
- Spatial correlation is also reproduced well.

Fig. 5

Combining SRE-EM with downscaling schemes to produce consistent space-time blended precipitation products

To address the issue of merging infrequent fine-resolution observations with frequent coarse-resolution observations to produce a consistent/reliable (in space and time) product, we performed a numerical experiment simulating three scenarios:

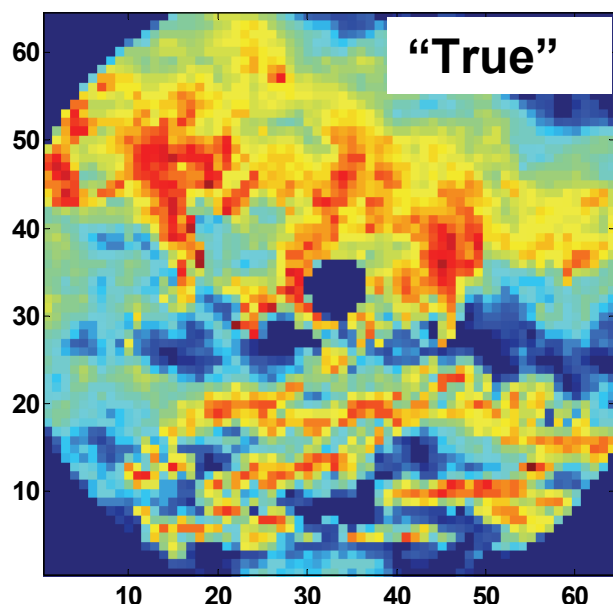
Case 1 is an idealized (best) case wherein observations are available at all times ($t = 1, 2$ and 3hr) at the coarse (16km) and fine (2km) resolutions.

Case 2 is a scenario in which high-resolution information (2km) is available only at $t = 1\text{hr}$ and the coarse resolution (16km) fields are available at all times ($t = 1, 2, 3\text{ hr}$). We perform a simple renormalization to match the total depth of water, i.e., $R_{2\text{km}}(t=2) = R_{2\text{km}}(t=1) * R_{16\text{km}}(t=2 \text{ or } 3\text{hr}) / R_{16\text{km}}(t=1)$, to obtain the 2 km field at $t = 2$ (or 3) hr.

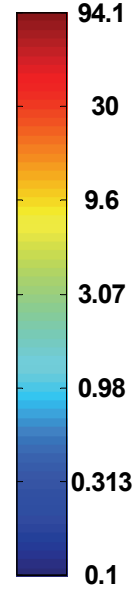
Case 3 is a scenario in which the available fields are the same as Case 2, except the low-resolution information (16km) at $t = 2$ and 3 hr is spatially downscaled to obtain a statistically consistent (moments, spatial correlation) fields at 2 km at $t = 2, 3\text{hr}$.

In all cases, 16 and 2 km fields (at $t = 1, 2$ and 3 hr) are input to SRE to obtain 4km fields at the respective times and these fields, in turn, are accumulated to obtain the 3-hr 4km fields

Downscaling (Case 3) improves the quality of the blended product

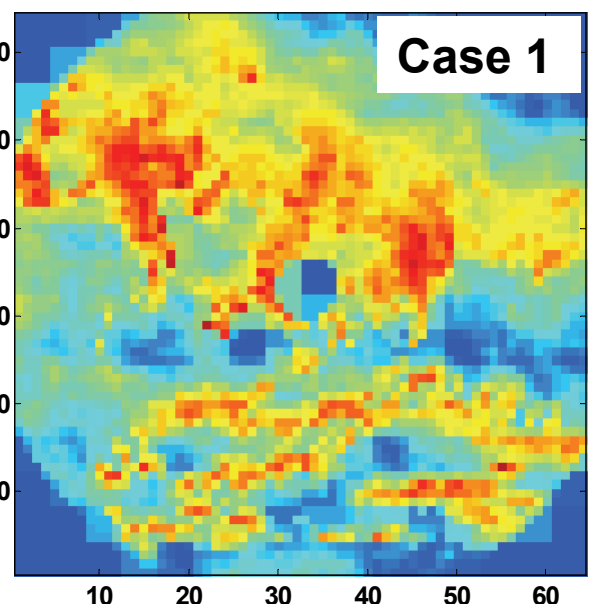


Without downscaling

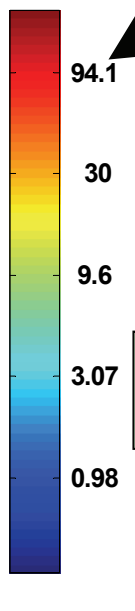
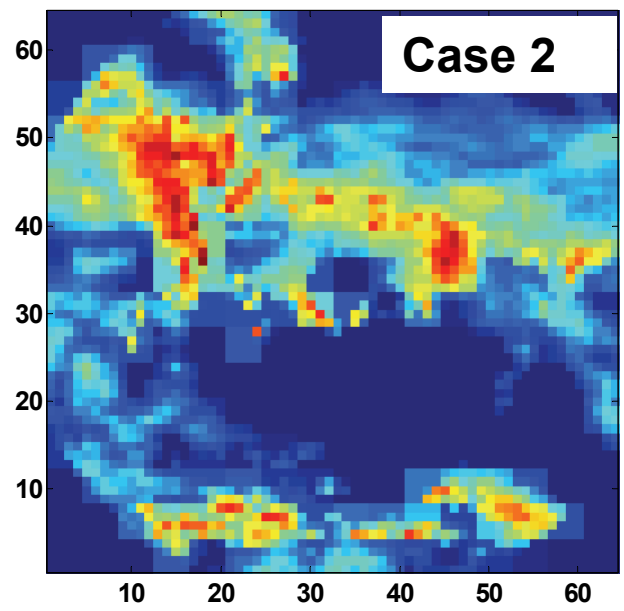
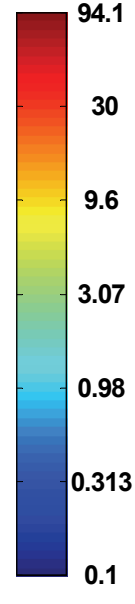


RMSE = 1.45
BIAS ~ 0.00

RMSE = 11.07
BIAS ~ 0.00



With downscaling



RMSE = 5.37
BIAS ~ 0.00

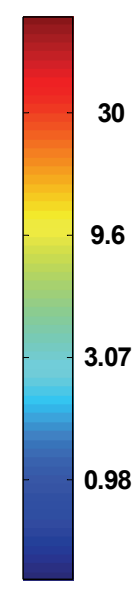
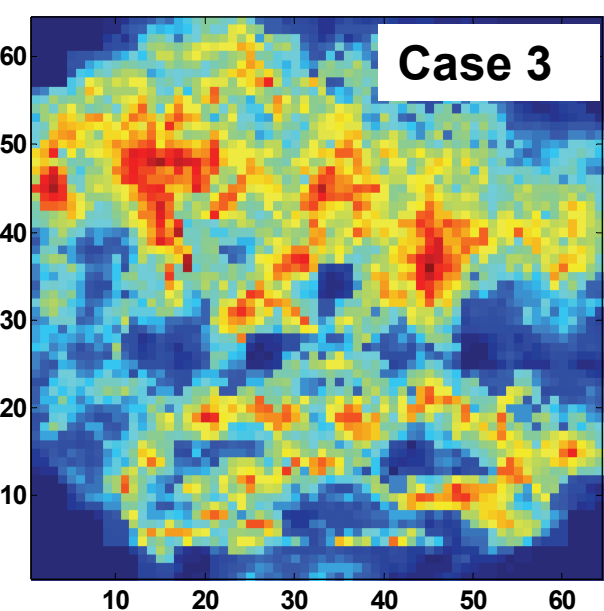


Fig. 7

Summary

- Scale-recursive estimation is an effective procedure to blend precipitation-related information from multiple sensors.
- Proposed Expectation-Maximization methodology addresses some of the drawbacks relating to the identification of the underlying multiscale structure of rainfall.
- Downscaling methodologies have the potential to be used in conjunction with merging procedures to obtain spatially and temporally “consistent” products.
- Future research will explore ways in which the proposed approach can be combined with a space-time downscaling scheme and/or temporal evolution of precipitation at a few locations only, such that the blended product at a desired scale incorporates all the available information in a space-time dynamically consistent way.

THE END

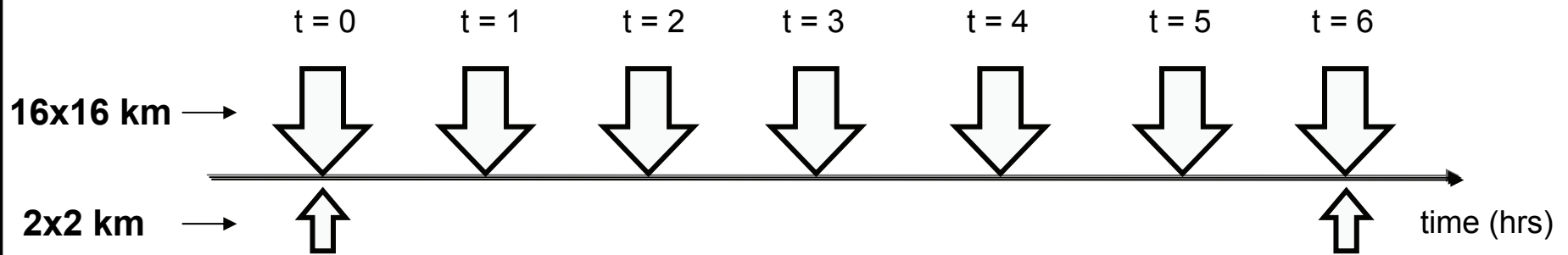
Discussion (Fig. 7)

- **Case 1** being the “best” case of available information, shows the least RMSE. This often is not realistically possible (*compare top left and top right images in Fig. 7*).
- **Cases 2 and 3** (with the same amount of information) represent more realistic scenarios in terms of available observations. However, the way we handle the information at the coarse scale dictates how well one can estimate an intermediate-scale field.
- In **case 2**, since such a simple renormalization does not take into account possibilities such as the “appearance” of cells (or, “disappearance”), the resulting 4km field is bound to be far from the “truth” (*compare top left and bottom left images in Fig. 7*)
- In **case 3**, we use the large-scale information effectively through a spatial downscaling scheme to resolve the small-scale variability. The visual comparison (*compare top left and bottom right images in Fig. 7*) of the observed and estimated (as well as quantitative comparisons such as the RMSE) is encouraging, and points to the advantages of using a spatial downscaling scheme.
- The spatial downscaling scheme used is based on our previous work, and its effectiveness is demonstrated separately in **Fig. 8**

Merging of infrequent high-resolution and frequent low-resolution precipitation observations

Given: 16 x 16 km precipitation estimates every 1 hour (e.g., satellite)

2 x 2 km precipitation estimates every 6 hours (e.g., radar)



Objective: 3-hr precipitation estimates at an intermediate scale of 4 x 4 km
along with their uncertainties

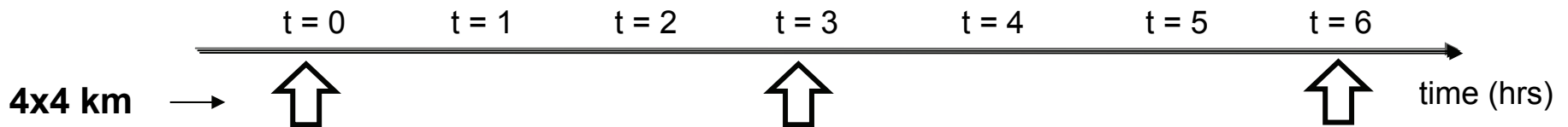
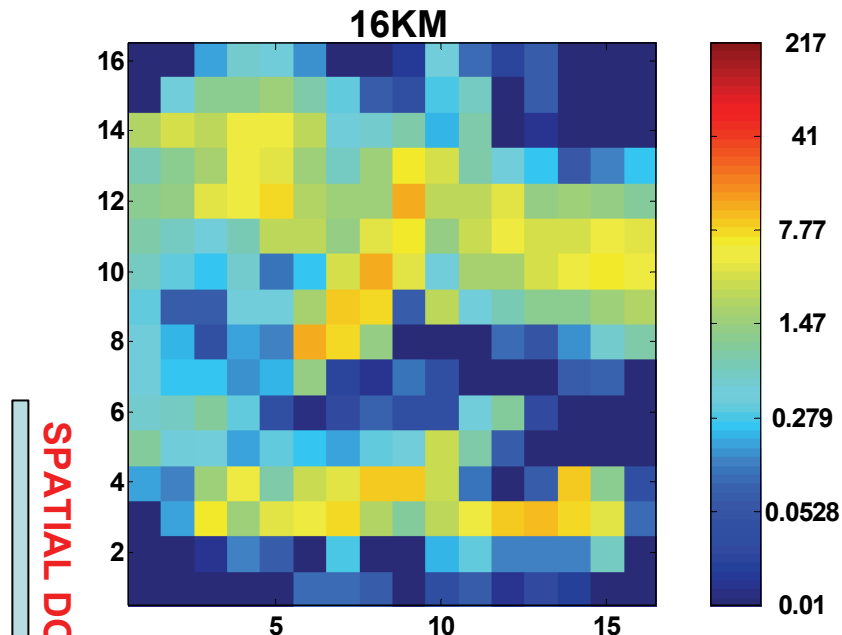


Fig. 6

Performance of the spatial downscaling scheme used in Case 3



The spatial downscaling scheme, which takes advantage of the presence of simple self-similarity in rainfall fluctuations, requires only one scaling parameter to resolve the small-scale variability (2km) given the large-scale variability (16km). This parameter has been found (Perica and Fofoula-Georgiou, JGR-Atmos. 101(D3); 101(D21), 1996) to be dependent on the available potential energy in the atmosphere (CAPE), and thus provide a viable (and parsimonious) means of resolving small-scale variability.

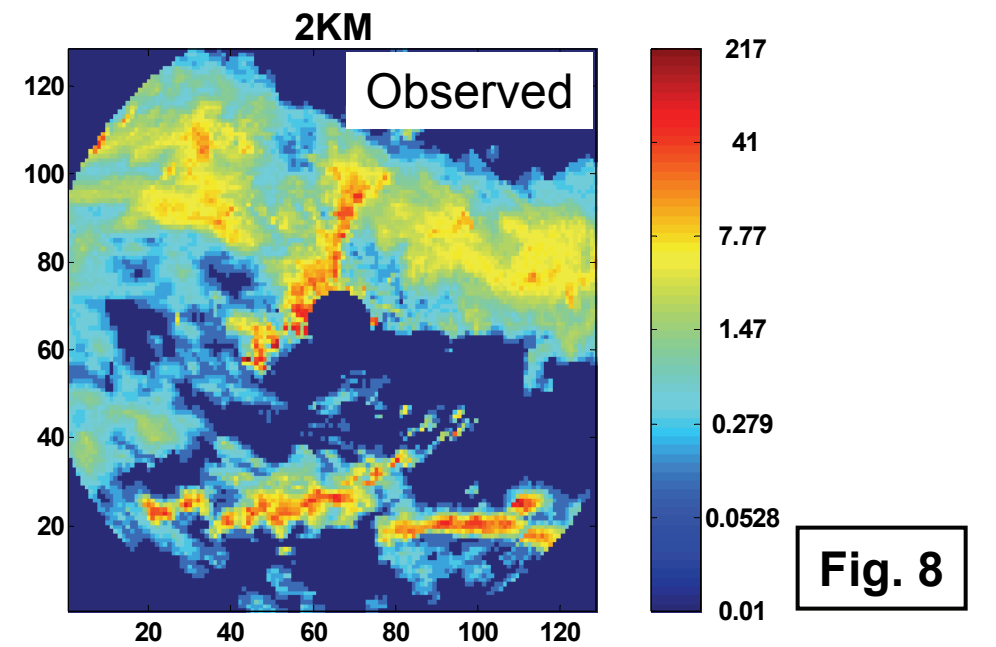
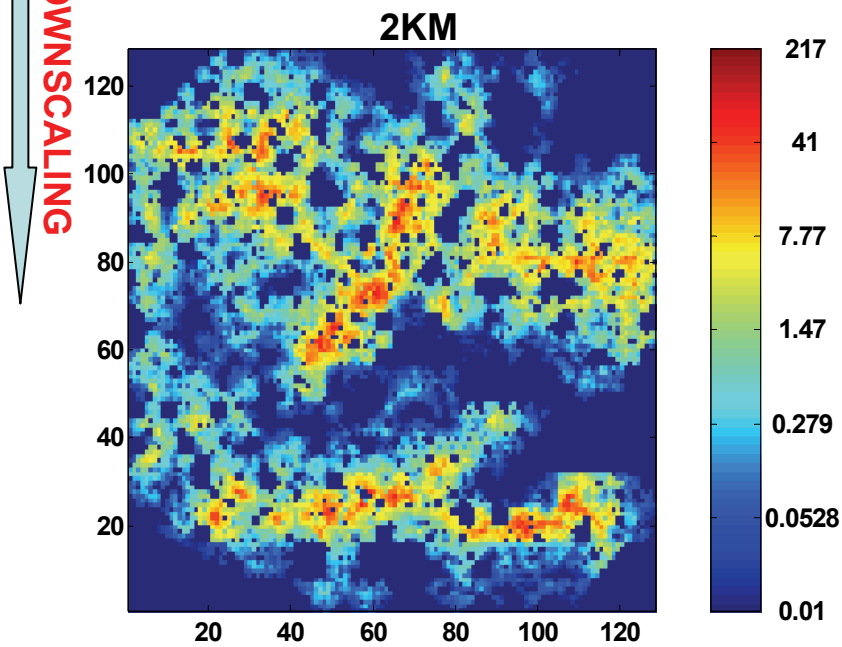
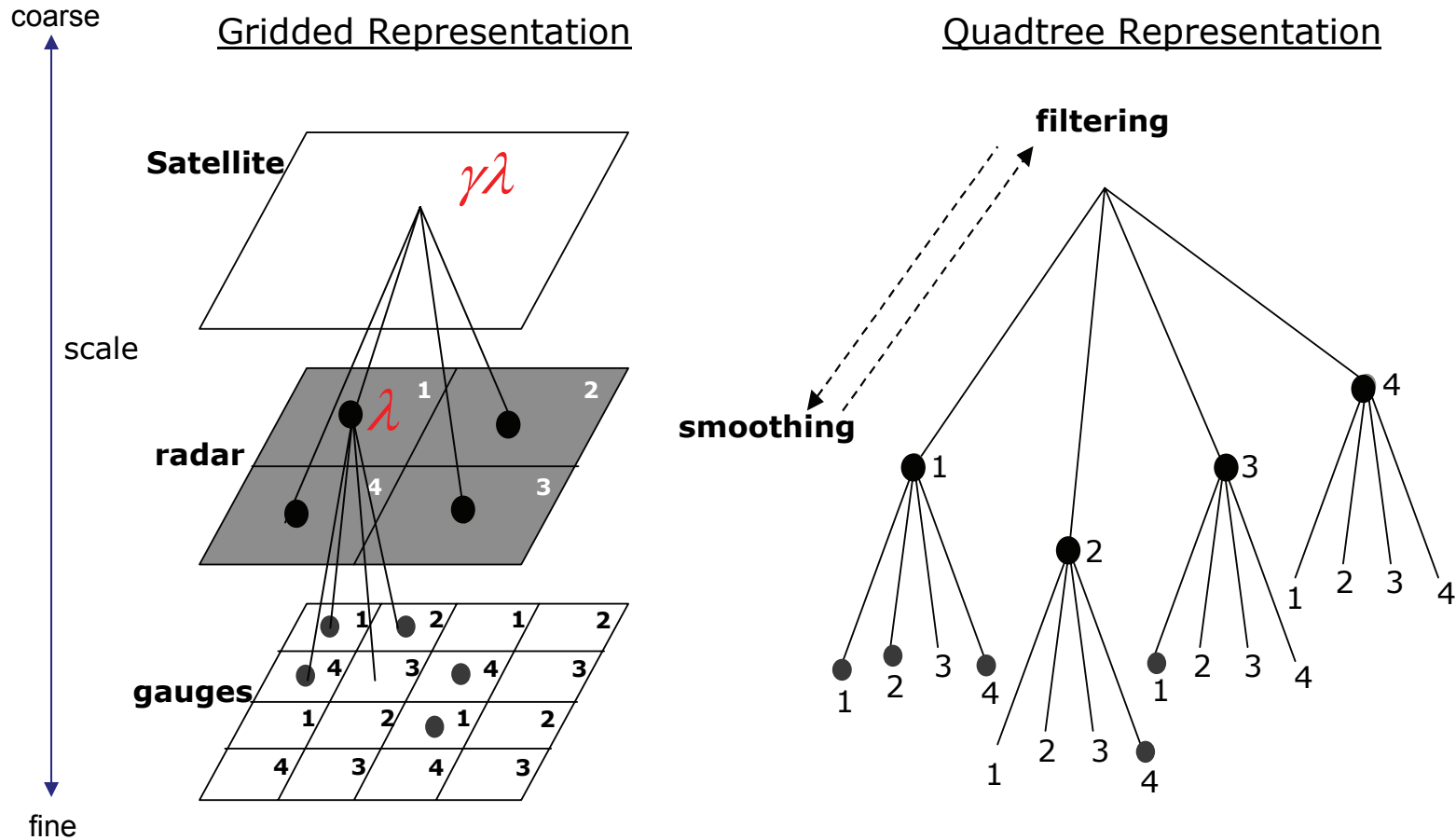


Fig. 8

Quadtree Representation of a Multiscale Process



Sparsely distributed measurements at the finer scale (gray dots), and measurements at a coarser scale (solid black dots), are placed on an inverted quadtree and merged via filtering and smoothing to obtain estimates at multiple scales

Fig. 2

SRE Formulation

State-space equation: relates the process state from one scale to another

$$X(\lambda) = A(\lambda)X(\gamma\lambda) + B(\lambda)W(\lambda), \quad W(\lambda) \sim N(0,1)$$

Measurement equation: relates process measurements and state

$$Y(\lambda) = C(\lambda)X(\lambda) + V(\lambda), \quad V(\lambda) \sim N(0,R(\lambda))$$

The parameter set of the multiscale state-space model is

$$\{A(\lambda), B(\lambda), C(\lambda), R(\lambda) \mid \lambda \in T, T \text{ is set of all the nodes in the tree}\}$$

If both measurements and the state of the system represent precipitation intensity, we can assume 'A' and 'C' to be unity. Therefore,

$$A(\lambda) = 1 \quad \forall \lambda \in T$$

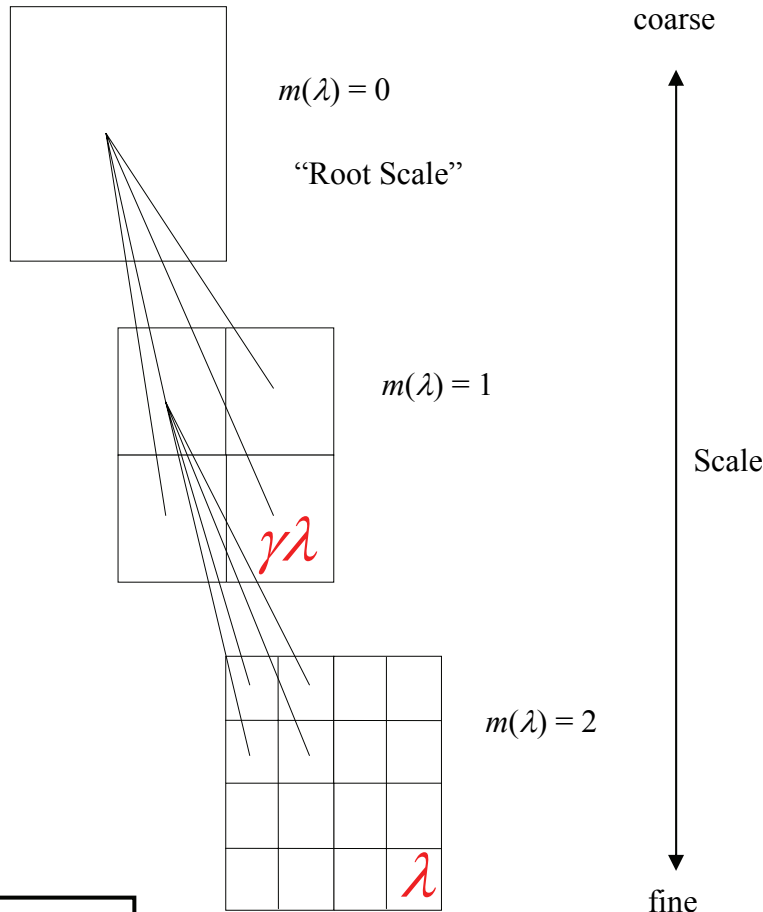
$$C(\lambda) = 1 \quad \forall \lambda \in T$$

$R(\lambda)$ = Measurement error variance
associated with each sensor

$B(\lambda), \forall \lambda \in T$, needs to be estimated

Estimation of Parameter $B(\lambda)$

➤ **Parametric Approach**: Multiplicative Cascade models $X(\lambda) = X(\gamma\lambda) W(\lambda)$



- **Log-normal cascade (LN)**:

$$w(\lambda) \sim e^{\sigma Z - \sigma^2/2}, \quad Z \sim N(0,1)$$

It can be shown that: $B(\lambda) = \sigma$

- **Bounded Log-normal cascade**: Weights lognormally distributed, but σ is chosen as a function of scale, λ :

- $\sigma_{bc}(\lambda) = \sigma_1 2^{-(m(\lambda)-1)H}$
- $\sigma_1 \equiv \sigma_{bc}(m(\lambda) = 1)$ and H controls the decay rate of the variance

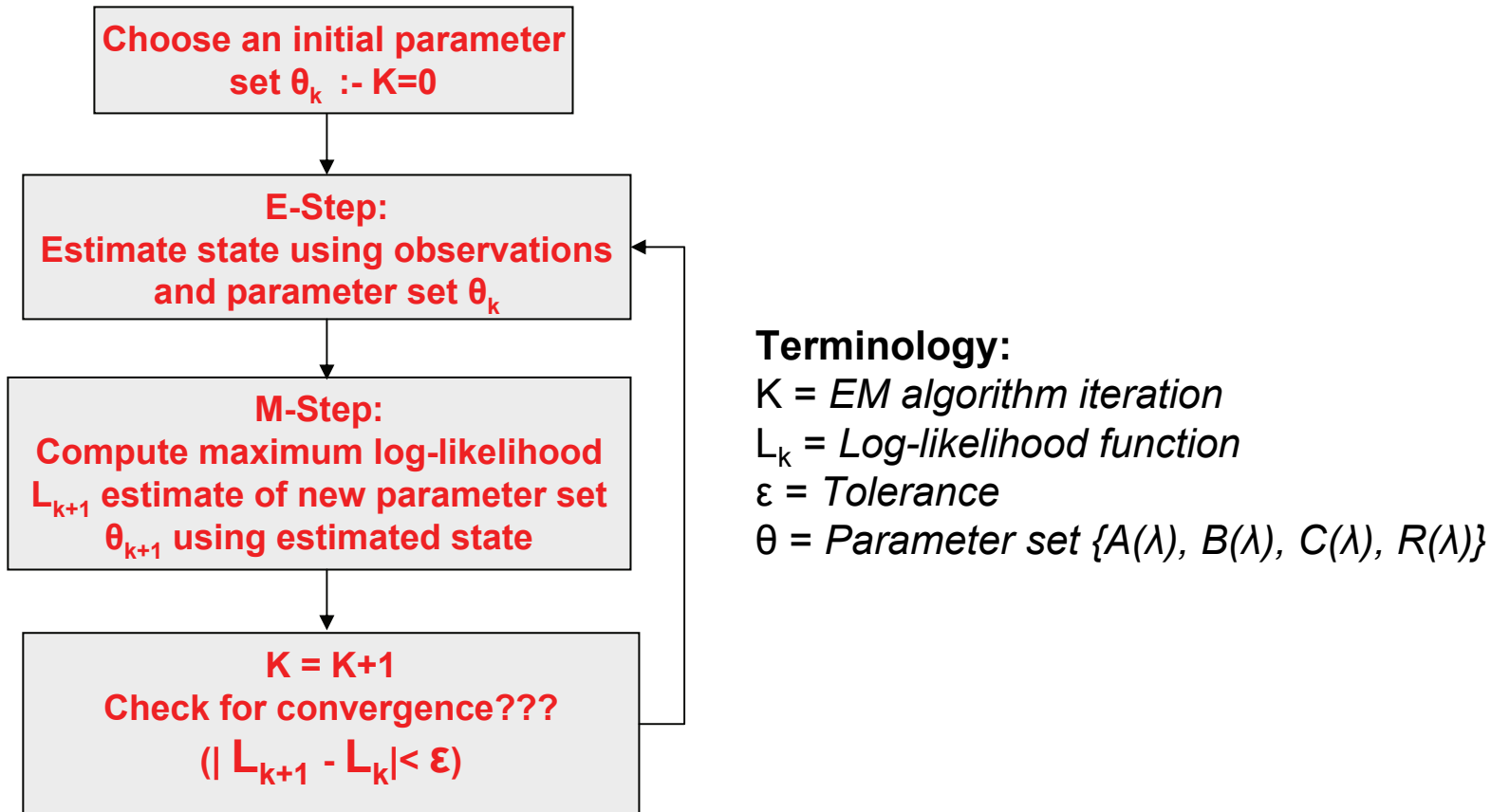
It can be shown that:

$$B(\lambda) = \sigma_{bc}(\lambda)$$

Fig. 4

Zero intermittency observed in precipitation presents a problem

EM algorithm – Flow Chart



See Appendix for details

Fig. 6

APPENDIX: Details of EM algorithm

- Log-likelihood function $L(X, Y, \theta)$ is given as follows

$$L(X, Y, \theta) = \sum_{\lambda \in (T-0)} \{ \log |B(\lambda)B^T(\lambda)| + [X(\lambda) - A(\lambda)X(\gamma\lambda)]^T [B(\lambda)B^T(\lambda)]^{-1} [X(\lambda) - A(\lambda)X(\gamma\lambda)] \} \\ - \sum_{\lambda \in T} \{ \log |R(\lambda)| + [Y(\lambda) - C(\lambda)X(\lambda)]^T R^{-1}(\lambda) [Y(\lambda) - C(\lambda)X(\lambda)] \} + \text{constant}$$

- The computation of log-likelihood function depends on three expectations

$$E[X(\lambda) | Y] \quad E[X(\lambda)X^T(\lambda) | Y] \quad E[X(\lambda)X^T(\gamma\lambda) | Y]$$

1. Kannan, A.et. al., "ML Parameter Estimation of a Multiscale Stochastic Process Using the EM Algorithm", IEEE Trans. Signal Processing, Vol. 48, No. 6, 2000.
2. M. R. Luettgen and A. S. Willsky, "Multiscale Smoothing Error Models", IEEE Trans. Automatic Control, Vol. 40, No. 1, 1995.

The Expectation Step (E-step)

- The E-Step of the algorithm computes the expected quantities required in the M-Step of EM algorithm

$$E\{X(\lambda) | Y_0\} = \hat{X}_s(\lambda)$$

$$E\{X(\lambda)X^T(\lambda) | Y_0\} = P_s(\lambda) + \hat{X}_s(\lambda)\hat{X}_s^T(\lambda)$$

$$E\{X(\lambda)X^T(\gamma\lambda) | Y_0\} = P_s(\lambda, \gamma\lambda) + \hat{X}_s(\lambda)\hat{X}_s^T(\gamma\lambda)$$

where $P_s(\lambda, \gamma\lambda) \cong E\{[X(\lambda) - \hat{X}_s(\lambda)][X(\gamma\lambda) - \hat{X}_s(\gamma\lambda)]^T | Y_0\}$

- The terms $\hat{X}_s(\lambda)$ and $P_s(\lambda)$ are computed by the downward sweep of the RTS algorithm
- $P_s(\lambda, \gamma\lambda)$ remains to be computed

Computation of $P_s(\lambda, \gamma\lambda)$

$P_s(\lambda, \gamma\lambda)$ is computed directly in the downward sweep by using the result that the smoothed error is a Gauss-Markov process

The smoothed error $\tilde{x}_s(\lambda) = x(\lambda) - \hat{x}_s(\lambda)$ has been shown to be modeled as a multiscale process of the following form

$$\tilde{x}_s(\lambda) = J(\lambda)\tilde{x}_s(\gamma\lambda) + \tilde{w}(\lambda)$$

$\tilde{w}(\lambda)$ is zero mean and white noise with covariance given by

$$P(\lambda | \lambda) - P(\lambda | \lambda)F^T(\lambda)P^{-1}(\gamma\lambda | \lambda)F(\lambda)P(\lambda | \lambda)$$

$$\begin{aligned} P_s(\lambda, \gamma\lambda) &= E\{\tilde{x}_s(\lambda)\tilde{x}_s^T(\gamma\lambda) | Y_o\} \\ &= E\{[J(\lambda)\tilde{x}_s(\gamma\lambda) + \tilde{w}(\lambda)]\tilde{x}_s^T(\gamma\lambda) | Y_o\} \\ &= J(\lambda)P_s(\gamma\lambda) \end{aligned}$$

The Maximization Step (M-step)

- This step maximizing the expected likelihood using multivariate regression to obtain new estimates of the parameters

$$\hat{A}(\lambda) = [E\{X(\lambda)X^T(\gamma\lambda)\}][E\{X(\gamma\lambda)X^T(\gamma\lambda)\}]^{-1}$$

$$\hat{B}(\lambda) = [E\{X(\lambda)X^T(\lambda)\} - \hat{A}(\lambda)E\{X(\gamma\lambda)X^T(\lambda)\}]^{1/2}$$

$$\hat{C}(\lambda) = [E\{Y(\lambda)X^T(\lambda)\}][E\{X(\lambda)X^T(\lambda)\}]^{-1}$$

$$\hat{R}(\lambda) = [E\{Y(\lambda)Y^T(\lambda)\} - \hat{C}(\lambda)E\{X(\lambda)Y^T(\lambda)\}]$$